MATH 1180
Midterm I

Do all three problems, points as indicated. Write readable answers on the test, but feel free to use or hand in additional paper if necessary. Remember, you can use notes and the book, but no calculator.

1. (35 points) A person decides to calibrate caloric intake $c(W)$ as a function of their weight W. In particular, suppose $c(W) = 3000 - 5W$, and that

$$\frac{dW}{dt} = c(W) - 10W.$$

  a. If $W(0) = 100$, find $c(W)$.

  b. If $W(0) = 100$, is weight increasing or decreasing?

  c. Find the equilibrium weight for this person.

  d. Draw a phase-line diagram.

  e. Sketch the graph of a solution starting from $W(0) = 100$.

2. (35 points) A more sophisticated dieter allows caloric intake C to follow its own differential equation.

$$\frac{dW}{dt} = C - 10W$$
$$\frac{dC}{dt} = \frac{300}{100 + W} - 1.$$

a. Draw the phase plane, replete with nullclines, equilibria and direction arrows.

b. Sketch a solution starting from a small weight and high number of calories.

3. (35 points) Eventually, people usually give up dieting and eat as much as they want. In this case, suppose that weight follows

$$\frac{dW}{dt} = \frac{300}{100 + W}.$$

Suppose that time is measured in weeks.

a. Suppose $W(0) = 100$. Use Euler's method to estimate $W(2)$.

b. Find an algebraic solution (it is tricky to find W as a function of t, feel free to quit).

c. Will it take more than a year for this person's weight to double?