MATH 1180
MATHEMATICS FOR LIFE SCIENTISTS
Computer Assignment IX
Due April 20, 2004

Warm up Maple for today's encounter with likelihood with the commands

```
> with(stats);
> iread(histplot);
> iread(binomial);
> iread(draw);
```

PROBLEMS

- **1.** You find that one out of 5 cells has a particular trait. Find and plot the likelihood function for this situation. Indicate the value of p giving the maximum likelihood. We would like to compare this likelihood function with samples of 100 drawn from binomial distributions with $n = 5$ and different values of p.

  To draw 100 numbers from a binomial distribution with $p = 0.2$ and count up the results, try

  ```
  > a := [seq(bindraw(5,0.2),i=1..100)]:
  > transform[tally](a);
  ```

  The colon at the end of the second line suppresses annoying output. Try with $p = 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4$ and 0.5 and mark the fraction of times you got exactly 1 success on your graph of the likelihood function.

  Use the support, the log of the likelihood function, to estimate the confidence limits from your original data (1 out of 5). Remember that the confidence limits are values of the parameter that give a support 2.0 less than the maximum. You will probably want to use `fsolve` to find the values (it is easiest to first plot a graph of the support along with a horizontal line that is 2.0 below the maximum). For extra fun, figure out how to find the exact confidence limits (values of p that have probability 0.025 of giving results as or more extreme than what you got). How do they compare with the values found with the method of support?

- **2.** Cells are placed for one minute in an environment where they are hit by X-rays, some of which are damaging. Cells not hit by the damaging rays are healthy, those hit exactly once are damaged and those hit more than once are dead. By measuring the states of a number of cells, we wish to infer the rate at which cells are hit by damaging rays.

  Let x denote the unknown parameter of the Poisson distribution. Use the formula for the Poisson distribution to compute the probabilities p_0 of no hits, p_1 of one hit and p_m of more than one hit in one minute as **functions** of x. Enter the arrays

```
> v := [0,1,2];
> p := [p0(x),p1(x),pm(x)];
```

  Suppose the true value of x is 3.0. Plot the histogram with the `histplot` command. Set a random seed `_seed` to a "random" value dredged from your unconscious. Sample 50 events using the `draw` command

```
> a := [seq(draw(v,p),i=1..50)];
```

  To keep things interesting, keep sampling until you get at least one cell of each type. Compare the results of your sample with the idealized histogram.

  We can use likelihood to analyze our data. Find the likelihood function L of this data (it is the product of the likelihoods for each of the 50 cells) as a function of y, and let S be the support, the natural log of L. Plot $S(y)$ over a reasonable range. Find the maximum with the following command and mark it on your graph:

```
> xmax := fsolve(diff(S(y),y),y);
```

  Find the support of the hypotheses $x =$`xmax`, $x = 2$, $x = 4$ and the "truth" $x = 3$, and indicate each on your graph. Mark the confidence limits on your graph.