MATH 1180
MATHEMATICS FOR LIFE SCIENTISTS
Computer Assignment X
Due April 27, 2004

Before opening Maple, set up the printer with the following command in your local window.

```
setenv PRINTER spps
```

Warm up Maple for today's problems with the commands

```
> with(stats);
> iread(histplot);
> iread(binomial);
> iread(draw);
```

PROBLEMS

- **1.** Recall the cells hit by X-rays from the previous computer lab. Those not hit by any damaging rays are healthy, those hit exactly once are damaged and those hit more than once are dead. Use the Poisson distribution to compute the probabilities $p_0(x)$ of no hits, $p_1(x)$ of one hit and $p_m(x)$ of more than one hit as functions of the Poisson parameter x. Enter the arrays

  ```
  > v := [0,1,2];
  > p := [p0(x),p1(x),pm(x)];
  ```

  Set $x = 3.0$. Pick a random seed `_seed` and simulate 25 cells with the command

  ```
  > a := [seq(draw(v,p),i=1..25)];
  > transform[tally](a);
  ```

  The `transform` command places the results into bins.

  We will test the hypothesis $x = 2.0$ using p-values. Figure out what might be deemed a more extreme result than what you found (one possibility is "more 2's than actually observed"). Let y represent a guess of the true parameter value. Use the cumulative binomial distribution to compute the probability of a more extreme result as a function h of y. Plot this function over a reasonable range and indicate the significance level of the test of the hypothesis $x = 2.0$. Is your data sufficient to reject this hypothesis?

  To find the lower confidence limit (the smallest value of y more or less consistent with the data when viewed in this way), solve

  ```
  > xstar := fsolve(h(y)=0.025,y);
  ```

  Set $x = x^*$. How many times do you think you'll have to generate arrays `a` as above to generate a result at least as extreme as your original result? Try it a few times.

- **2.** To generate 12 independent random numbers from a normal distribution with mean 10 and variance of 9, type

```
> a := [stats[random,normald[10,3]](12)];
```

The parameters in `normald[10,3]` are, respectively, the mean of 10 and, confusingly, the **standard deviation** of 3.

Maple will cheerfully find the average of your data with

```
> muhat := describe[mean](a);
```

If your value of `muhat` is within 0.5 of 10.0, try again to guarantee nice pictures and interesting results (real scientists are not allowed to do this sort of thing). The average of your 12 measurements comes from a normal distribution with mean 10 and some standard error. Find the standard error and the 95% confidence interval around `muhat`, calling the lower limit `mulow` and the upper limit `muhi`.

The goal in this problem is to simulate three experiments 100 times: averaging 12 values from normal distributions with true mean 10.0, true mean `mulow` and true mean `muhi`.

The averages of 100 experiments with true mean 10 can be simulated with

```
> ave := [seq(describe[mean]([stats[random,normald[10,3]](12)]),j=1..100)];
```

Spend at least 7 seconds thinking about what this command does. A relative of `tally` designed for continuous distributions is `tallyinto`. The command

```
> transform[tallyinto](ave,[0..mulow,mulow..10,10..muhat,muhat..muhi,muhi..30]);
```

tells how many values in `ave` lie below `mulow`, between `mulow` and 10.0, etc (if your value of `muhat` lies below 10.0, switch the order accordingly). The value 30 was chosen to be sure to be larger than any possibility. Create arrays `avelow` and `avehi` similar to `ave` with means `mulow` and `muhi` instead of `10`. Run your results through `tallyinto`.

Define functions `f`, `fl` and `fh` to be the p.d.f.'s describing the distribution of elements in `ave`, `avelow` and `avehi`, and `fhat` with mean `muhat`. Print a graph of all four functions, and list below them the number of elements of `ave`, `avelow` and `avehi` lying in the various intervals. Indicate which ones have values predicted by the theory of confidence intervals and what those values should be. Are you bothered by the fact that more than 5% of the elements of `ave` lie outside your confidence interval?