Chapter 6

Revised Exercises

6.1 Introduction to Probabilistic Models

MATHEMATICAL TECHNIQUES

♠ In each of the following populations, per capita reproduction changes over time according to a fixed pattern (rather than a random pattern). For each population, find the population during the first six years and sketch a graph. Do you think it will grow or shrink in the long run? Find the average per capita reproduction in each case. Does an average greater than 1.0 always mean that the population is growing?

- EXERCISE **6.1.1**

  A population starts at size 1000 and grows with per capita reproduction $r(t)$ that alternates between 0.6 and 1.5.
- EXERCISE **6.1.2**

  A population starts at size 1000 and grows with per capita reproduction $r(t)$ that alternates between 0.7 and 1.5.
- EXERCISE **6.1.3**

  A population starts at size 1000 and grows with per capita reproduction $r(t)$ that has a three year cycle, first 0.7, then 0.9, and then 1.6.
- EXERCISE **6.1.4**

  A population starts at size 1000 and grows with per capita reproduction $r(t)$ that has a three year cycle, first 0.7, then 0.9, and then 1.5.

♠ In each of the following populations, immigration changes over time according to a fixed pattern (rather than a random pattern). For each population, find the population in each of the first 6 years and sketch a graph. Do you think it will grow or shrink in the long run? Find the average immigration in each case. Does an average greater than zero mean that the population is growing?

- EXERCISE **6.1.5**

  A population starts at size 100, and receives 10 immigrants in the first year, loses 5 emigrants in the second year, receives 10 immigrants in the third year and so forth.
- EXERCISE **6.1.6**

  A population starts at size 100, and receives 10 immigrants in the first year, loses 12 emigrants in the second year, receives 10 immigrants in the third year and so forth.
- EXERCISE **6.1.7**

  A population starts at size 50, and receives 10 immigrants in the first year, loses 12 emigrants in the second year, and gains 5 immigrants in the third year. It then repeats this three year cycle.
- EXERCISE **6.1.8**

  A population starts at size 20, and has a six year cycle: gain 1, lose 2, gain 3, lose 4, gain 3, lose 2.

♠ The following table gives the per capita reproduction for 4 populations over a period of 10 years. Find the population over the ten years starting from the given initial population, sketch a graph, and check whether the population has increased or decreased. Find the average per capita reproduction (add up the 10 values for per capita reproduction and divide by 10). Does an average greater than one always mean that the population is growing?

Year	Growth of population 1	Growth of population 2	Growth of population 3	Growth of population 4
1	1.030	0.670	0.960	0.997
2	0.886	0.870	0.841	1.030
3	0.564	1.020	1.450	1.100
4	1.050	1.480	0.966	1.040
5	0.507	0.602	1.260	1.110
6	0.919	0.941	0.769	0.991
7	0.632	0.911	1.270	1.020
8	0.712	1.350	0.967	1.110
9	1.360	0.883	1.180	0.935
10	1.250	1.420	0.883	0.958

- EXERCISE **6.1.9**
  Population 1 starting from 100 individuals.
- EXERCISE **6.1.10**
  Population 2 starting from 50 individuals.
- EXERCISE **6.1.11**
  Population 3 starting from 500 individuals.
- EXERCISE **6.1.12**
  Population 4 starting from 200 individuals.

♠ The following table gives the immigration (positive values) or emigration (negative values) for 4 populations over a period of 10 years. Find the population after 10 years starting from the given initial population, and check whether the population has increased or decreased. Find the average immigration (add up the 10 values and divide by 10). Does an average greater than zero mean that the population is growing?

Year	Change in population 1	Change in population 2	Change in population 3	Change in population 4
1	2	1	1	0
2	-1	3	-1	3
3	1	-1	0	2
4	-2	3	1	-2
5	-3	0	-3	3
6	-2	0	-1	1
7	-2	4	-3	-3
8	3	-3	1	0
9	0	-1	0	1
10	4	3	3	-1

- EXERCISE **6.1.13**
  Population 1 starting from 10 individuals.
- EXERCISE **6.1.14**
  Population 2 starting from 20 individuals.
- EXERCISE **6.1.15**
  Population 3 starting from 10 individuals.
- EXERCISE **6.1.16**
  Population 4 starting from 50 individuals.

♠ Describe what would happen to a population following the Markov chain for occupation and extinction (equations 6.3 and 6.4) in the following special cases.

- EXERCISE **6.1.17**
  The probability of an empty island being occupied and the probability of an occupied island becoming empty are both equal to 1.

• EXERCISE **6.1.18**
The probability of an empty island being occupied is 0 and the probability of an occupied island becoming empty is 0.5.

APPLICATIONS

♠ The following table describes populations that are growing through reproduction. For each, sketch a graph, find the per capita reproduction in each year, and describe the growth of the population in words.

Year	Population 1	Population 2	Population 3	Population 4
1	100	100	100	100
2	110	110	98	84
3	121	124	103	66
4	133	134	118	102
5	146	150	135	151
6	161	167	161	144
7	177	181	153	201
8	195	200	166	174
9	214	222	183	278
10	236	249	230	160
11	259	277	235	112

• EXERCISE **6.1.19**
What is the per capita reproduction of population 1 during these years? When will the population reach 500?

• EXERCISE **6.1.20**
What is the per capita reproduction of population 2 during these years? When will the population reach 500?

• EXERCISE **6.1.21**
What is the per capita reproduction of population 3 during these years? When will the population reach 500?

• EXERCISE **6.1.22**
What is the per capita reproduction of population 4 during these years? When will the population reach 500?

♠ The following table describes populations that are growing through immigration or emigration. For each, sketch a graph, find the number of individuals that arrived or left in each year, and describe the growth of the population in words.

Year	Population 1	Population 2	Population 3	Population 4
1	50	50	50	50
2	52	51	54	54
3	54	52	56	59
4	56	54	60	59
5	58	57	62	63
6	60	59	64	65
7	62	62	68	72
8	64	65	73	69
9	66	66	75	74
10	68	67	77	70
11	70	69	79	70

• EXERCISE **6.1.23**
What is the change in population 1 during these years? When will the population reach 100?

• EXERCISE **6.1.24**
What is the change in population 2 during these years? When will the population reach 100?

• EXERCISE **6.1.25**
What is the change in population 3 during these years? When will the population reach 100?

• EXERCISE **6.1.26**
What is the change in population 4 during these years? When will the population reach 100?

♠ Suppose the state of populations on four islands are described in the following table.

Year	Island 1	Island 2	Island 3	Island 4
0	Occupied	Occupied	Occupied	Occupied
1	Extinct	Occupied	Occupied	Extinct
2	Occupied	Extinct	Occupied	Occupied
3	Extinct	Occupied	Extinct	Occupied
4	Occupied	Occupied	Occupied	Extinct
5	Occupied	Extinct	Occupied	Occupied
6	Occupied	Occupied	Extinct	Extinct
7	Extinct	Occupied	Occupied	Occupied
8	Occupied	Extinct	Occupied	Extinct

For each, illustrate what is happening with a graph, and describe it in words. Does any of the islands have a pattern that can be described deterministically?

- EXERCISE **6.1.27**

  On island 1.

- EXERCISE **6.1.28**

  On island 2.

- EXERCISE **6.1.29**

  On island 3.

- EXERCISE **6.1.30**

  On island 4.

♠ The simple models of stochasticity described in the book leave out a lot of biological detail. Use your imagination to add some of that detail back.

- EXERCISE **6.1.31**

  Think of two biological factors that are neglected in the stochastic model $b_t = r(t)b_t$, equation 6.1.

- EXERCISE **6.1.32**

  Think of three factors neglected in the stochastic model of immigration (equation 6.2).

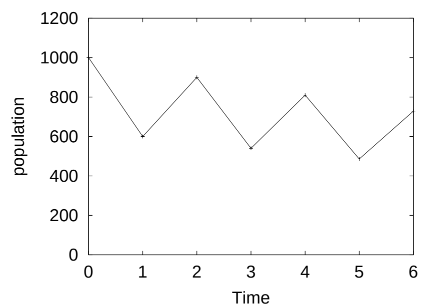
- EXERCISE **6.1.33**

  Think of two factors neglected in the Markov chain model of presence and absence on a island (equations 6.3 and 6.4).

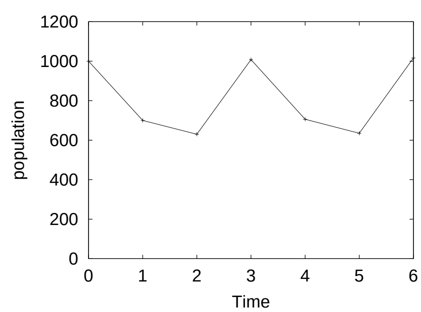
Chapter 7

Answers

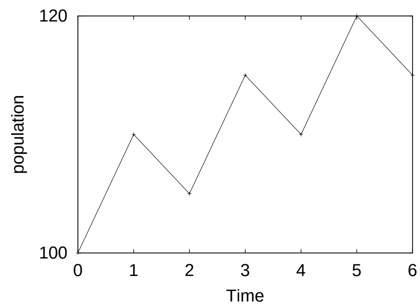
6.1.1. The population will be 600 after one generation, 900 after two, 540 after three, 810 after four, 486 after five and 729 after six. This population is shrinking. This is surprising because the average per capita reproduction is $(0.6 + 1.5)/2 = 1.05 > 1$.



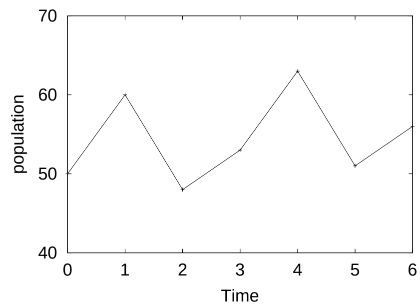
6.1.3. The population will be 700 after one generation, 630 after two, 1008 after three, 706 after four, 635 after five, and 1016 after six. This population is growing, but very slowly. The average per capita reproduction is $(0.7 + 0.9 + 1.6)/3 = 1.067 > 1$.



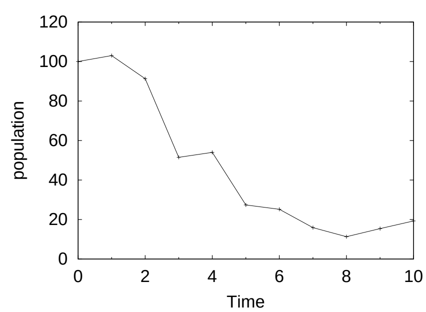
6.1.5. The population is 110, 105, 115, 110, 120, 115 during the first six years. It is growing, consistent with the fact that the average number of immigrants is $(10 + (-5))/2 = 2.5$.



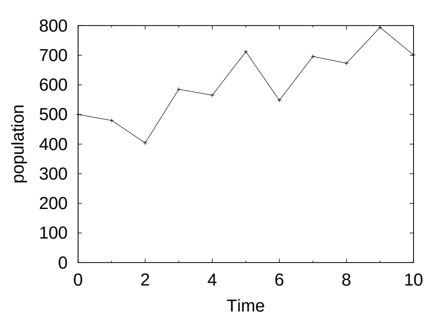
6.1.7. The population is 60, 48, 53, 63, 51, 56 during the first six years. It is growing, consistent with the fact that the average number of immigrants is $(10 + (-12) + 5)/3 = 1$.



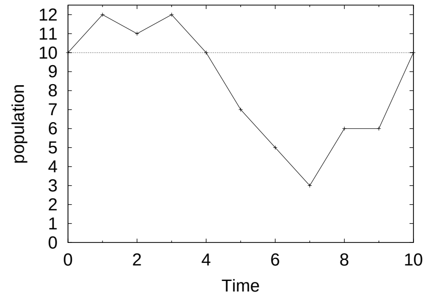
6.1.9. The population after 1 generation is $1.03 \cdot 100 = 103$, and after 2 generations is $0.886 \cdot 103 = 91.258$ and so forth. After 10 generations, the population is 19.262. The average per capita reproduction is 0.891, which is less than 1.0, consistent with the fact that this population shrinks.



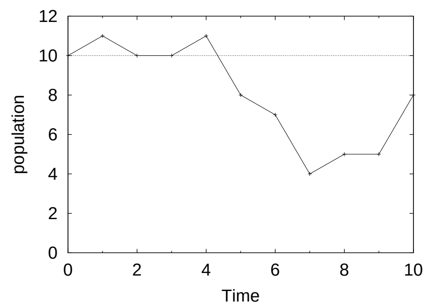
6.1.11. After 10 generations, the population is 701.055. The average per capita reproduction is 1.055, which is greater than 1.0. This is consistent with the fact that this population grows.



6.1.13. After one year, the population is $10+2 = 12$, and after two years it is $12-1 = 11$ and so forth. Continuing in this way, we find that the population after 10 years is still 10, unchanged. The average immigration is exactly 0, consistent with the lack of change.

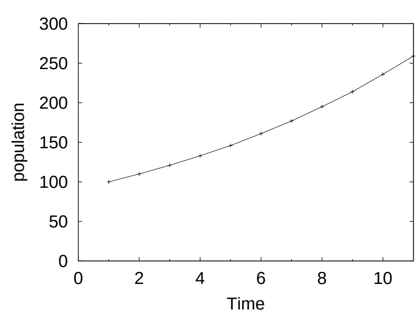


6.1.15. The population after 10 years is 8. The average immigration is -0.2, consistent with the decrease in population.

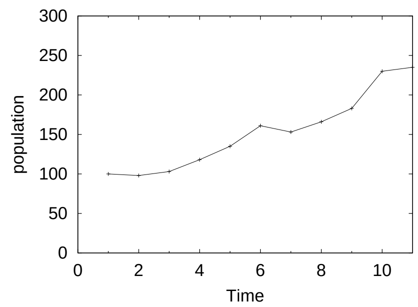


6.1.17. The island would jump back and forth between being empty and being occupied, switching every year.

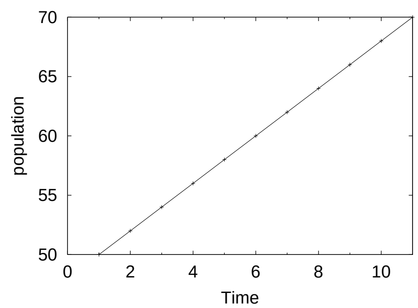
6.1.19. Between years 1 and 2, the per capita reproduction is the ratio of the population sizes, or 111/100=1.1. The same hold during the next interval and so on. This population is growing deterministically with per capita reproduction 1.1. It will reach 500 when $100 \cdot 1.1^t = 500$, or $1.1^t = 5$, or $t = \ln(5)/\ln(1.1) = 16.9$, or after 17 years.



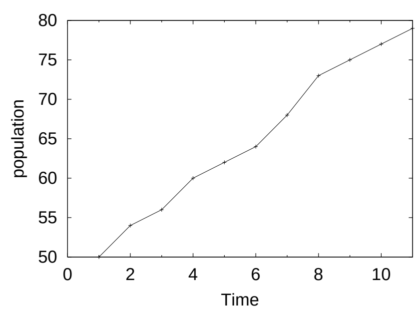
6.1.21. During the first 10 years, the per capita reproduction is 0.98, 1.05, 1.15, 1.14, 1.19, 0.95, 1.08, 1.10, 1.26 and 1.02. These values average 1.092, and take on values fairly close to 1.10 every year, but sometimes are less than 1. I would guess it might take longer than 17 years to reach 500, because the bad years slow it down.



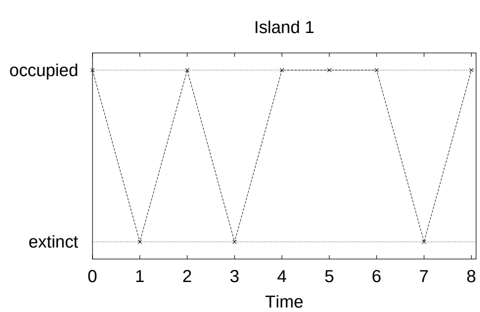
6.1.23. During the first 10 years, the change is 2 every year. It would take exactly 25 years to reach 100.



6.1.25. During the first 10 years, the change is 4, 2, 4, 2, 2, 4, 5, 2, 2 and 2. These average 2.9 per year, and range from 2 to 5. It would take about 50/2.9=17.2 years to reach 100.

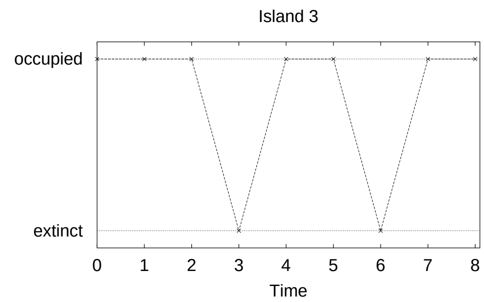


6.1.27.



This island jumps back and forth, but seems to be occupied more often than unoccupied.

6.1.29.



This island tends to be occupied for a few years and then extinct for 1 year.

6.1.31. 1. The per capita reproduction does not depend on the population size. 2. There is no immigration or emigration.

6.1.33. 1. It doesn't take into account the size of the population. 2. It doesn't take into account other nearby islands that might also be occupied or unoccupied.