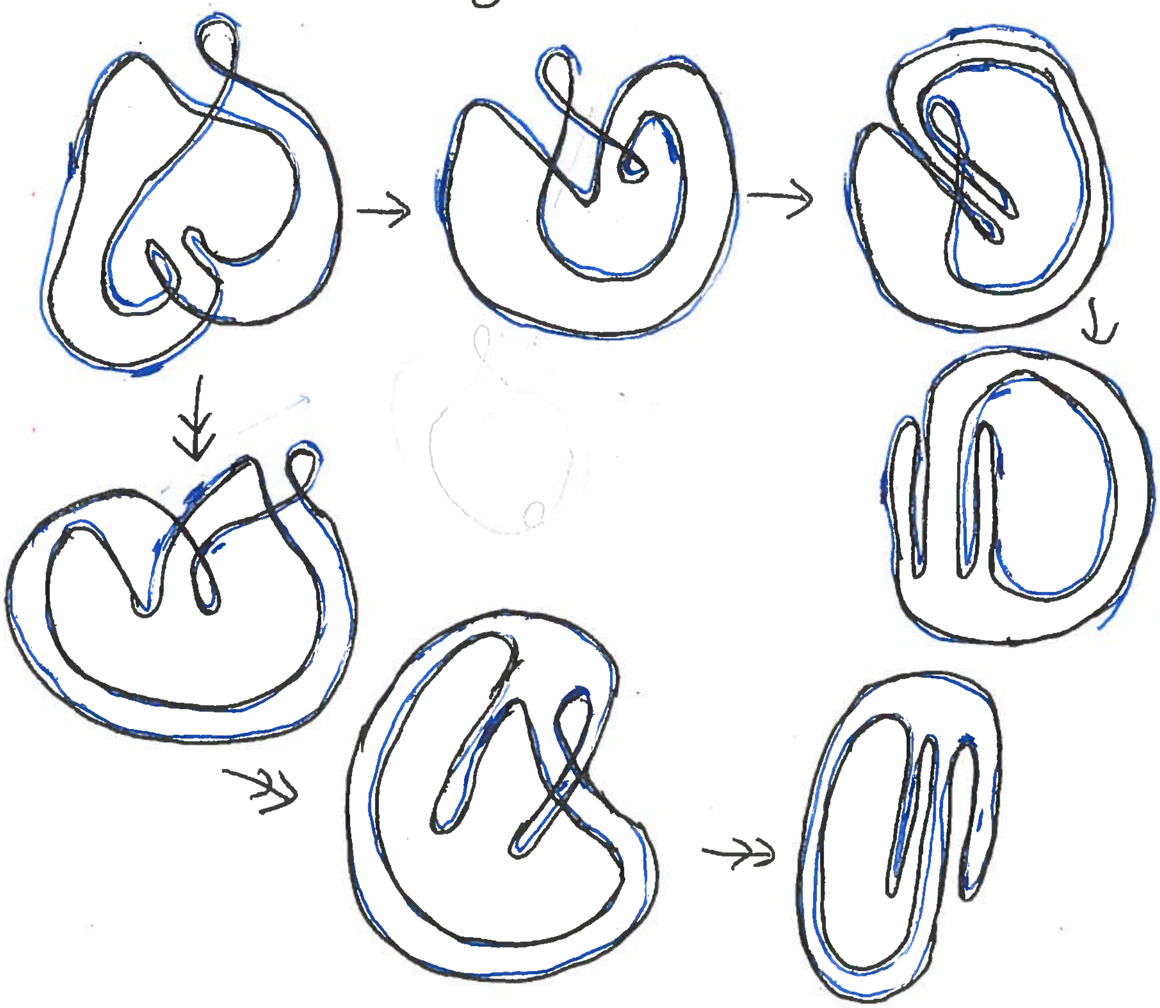
Derek Hacon

<u>Torus eversion</u> comes from disentangling immersed torus in two ways:



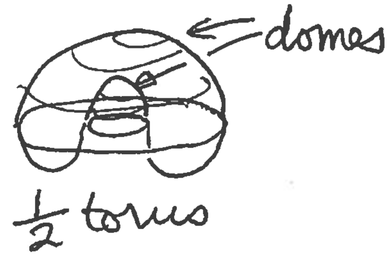
putting these together* gives eversion. Observe deformation is symmetric

(* : clockwise)

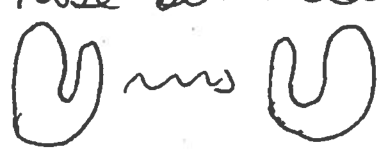
with respect to horizontal plane $z=0$.

<u>Sphere eversion</u> replace top half of torus eversion by two domes which meet the plane in the given curves: i.e. movement of domes follows movement of curves:

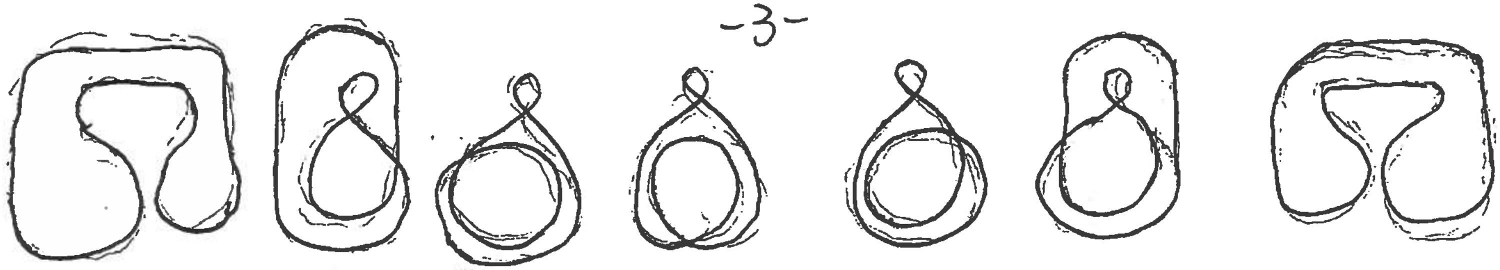
(sphere $= \frac{1}{2}$ torus $+ 2$ domes)



Movement of curves in plane is:

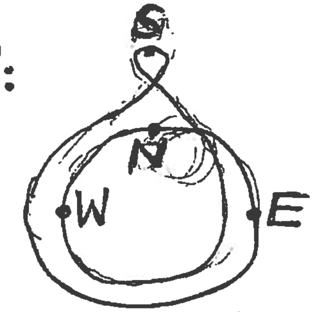
Easy to imagine a family of (embedded) domes whose boundaries are the <u>first set</u>: 

Redraw <u>second</u> sequence as:

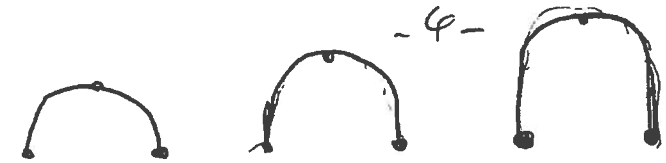


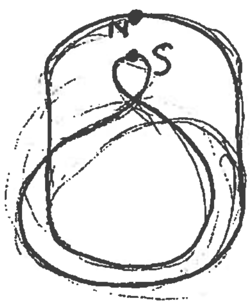
(Sequence is symmetric about the middle)

In the middle three, the inner kink travels around the circle (from W to E). Next, do a Northwards stretch, finally an Eastwards one. We choose points N E S W on the curve where the normal vector points N, E, S, W:

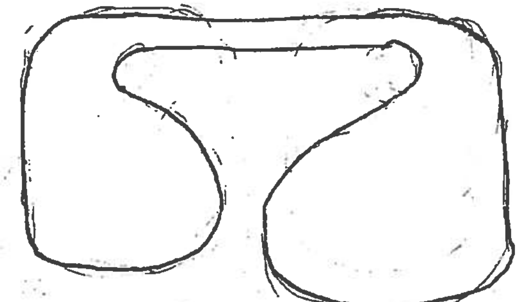


In the Northwards stretch, the WNE part moves

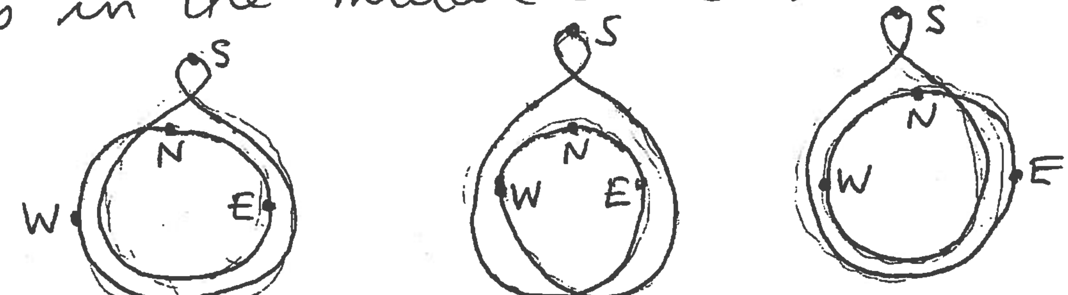
Northwards:  until it passes over the upper kink . Thus the points W and E expand into vertical intervals .

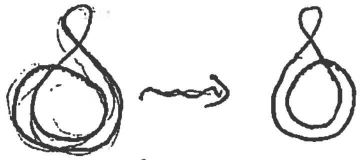
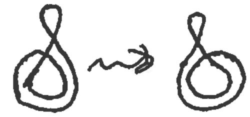
We get 

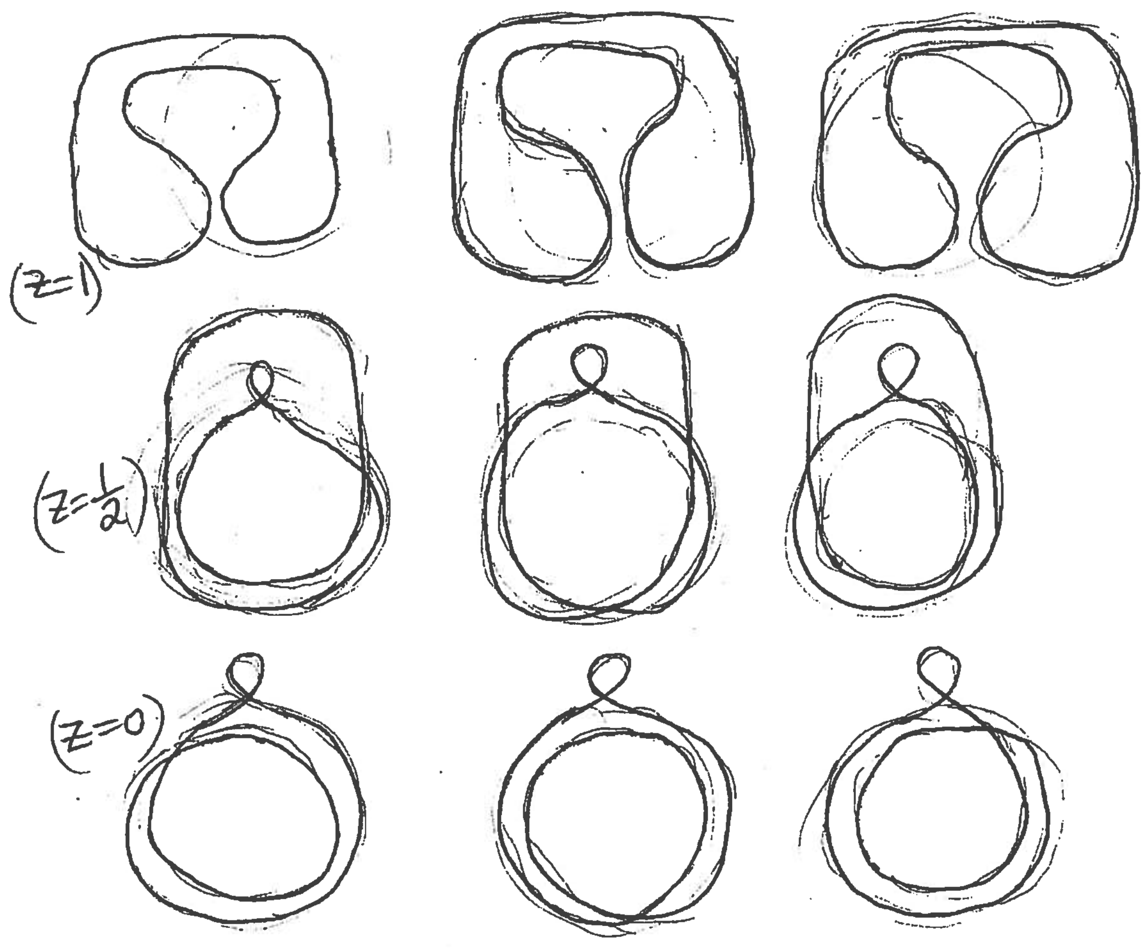
. Next, an <u>Eastwards stretch</u> gives...



The same procedure can be applied to all curves in the middle three section



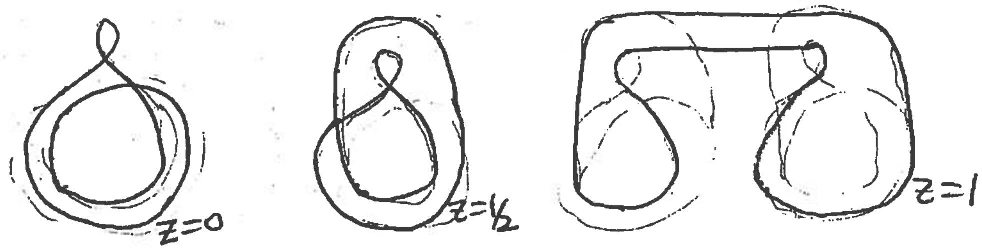
So this family 
can be simultaneously disentangled,
giving a family of immersed
cylinders with base curves 



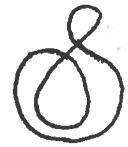
(level curves of immersed cylinders)

Sticking roofs on the top of this family of immersed cylinders

we get a family of immersed domes. Consider the last immersed dome whose cross-sections are



Imagine it slowly sinking. We get a family of immersed domes whose base curves are the above cross-sections, ending up with an embedded dome with base .

The domes for the initial part are similar but, of course, in the opposite order. So, first we see an embedded dome  with base  which rises upwards until its base is , then it is deformed into a dome with base  which, finally, sinks down into an embedded dome, with base 

For the other (embedded) set of curves the cylinders are not necessary and we can just erect (in our heads, at least!) the corresponding domes. Since the two curves are disjoint at the beginning and the end we can make the two embedded domes disjoint (at beginning and end).

Putting the pieces together gives a sphere eversion