(1) Compute all of the powers of 2 in $\mathbb{F}_{13}$.

(2) Find the order of each element of $\mathbb{F}_{13}^*$.

(3) Let $I(x)$ be the discrete logarithm in base 2 (which in this case is the inverse of $f(x) = 2^x$ modulo 13). Compute $I(11)$.

(4) Use the discrete logarithm to solve $x^7 = 5$ modulo 13.

The powers of 2 are $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 3$, $2^5 = 6$, $2^6 = -1$, $2^7 = -2 = 11$, $2^8 = -4 = 9$, $2^9 = -8 = 5$, $2^{10} = -3 = 10$, $2^{11} = -6 = 7$, $2^{12} = 1$.

The elements of order 1, 2, 3, 4, 6, 12 respectively are $\{1\}$, $\{-1\}$, $\{3, 9\}$, $\{8, 5\}$, $\{4, 10\}$, $\{2, 6, 11, 7\}$.

Since $2^7 = 11$, we have $I(11) = 7$.

Taking the discrete logarithm of $x^7 = 5$ we have $7 \cdot I(x) = I(5) = 9$ (modulo 12). Since $-5 \cdot 7 = -35 \equiv_{12} 1$, we have $I(x) = -5 \cdot 7 \cdot I(x) = -5 \cdot 9 = -45 \equiv_{12} 3$ and so $x = 2^3 = 8$. 

Calculators, cell phones, or notes are not allowed. The problems are worth 5 pts.