(1) Find all solutions to $35x + 40 = 0$ in $\mathbb{Z}/50\mathbb{Z}$.

$5(7x + 8) \equiv_{50} 0$ is equivalent to $(7x + 8) \equiv_{10} 0$ i.e. $7x \equiv_{10} 2$. Since $3 \cdot 7 = 21 \equiv_{10} 1$, we have $x \equiv_{10} 3 \cdot 7x \equiv_{10} 3 \cdot 2 = 6$. Thus $x = 6, 16, 26, 36, 46$ are all possible solutions.

(2) Show that $\mathbb{F}_{13}[i]$ is not a field and find the inverse of $3 + 5i$ in $\mathbb{F}_{13}[i]^*$.

Since $13 = 2^2 + 3^2$, we have $(2 + 3i)(2 - 3i) = 0$ so $2 + 3i, 2 - 3i$ are 0 divisors and hence $\mathbb{F}_{13}[i]$ is not a field.

On the other hand $3^2 + 5^2 = 34 \equiv_{13} 8$ is invertible and $8 \cdot 5 = 40 \equiv_{13} 1$. Thus $(3 + 5i)^{-1} = (3 - 5i) \cdot 5 = 15 - 25i = 2 + i.$