Math 4400, Fall 2014 Quiz 11

Name: ________________________________

Calculators, cell phones, or notes are not allowed.

(1) Factor $8 + 11i$ in to indecomposable elements of $\mathbb{Z}[i]$.

$N(8 + 11i) = 8^2 + 11^2 = 64 + 121 = 185 = 5 \cdot 37$. Thus

$$(8 + 11i)(8 - 11i) = (1 + 2i)(1 - 2i)(6 + i)(6 - i),$$

where $6 \pm i$ and $1 \pm 2i$ are irreducible (as $6^2 + 1^2 = 37$ and $1^2 + 2^2 = 5$ are primes congruent to 1 modulo 4). Next, we have

$$\frac{8 + 11i}{1 + 2i} = \frac{(8 + 11i)(1 - 2i)}{(1 + 2i)(1 - 2i)} = \frac{30 - 5i}{5} = 6 - i$$

and so $(8 + 11i) = (1 + 2i)(6 - i)$ is the required factorization.

(2) Given that $23^2 + 6^2 = 5 \cdot 113$ and 113 is prime, find $a, b$ such that $a^2 + b^2 = 113$ (Hint: you may want to find the gcd between 113 and a Gaussian integer of norm equal to $23^2 + 6^2$).

We divide 113 by $23 + 6i$ to get

$$\frac{113}{23 + 6i} = \frac{113(23 - 6i)}{(23 + 6i)(23 - 6i)} = \frac{113 \cdot 23 - 113 \cdot 6i}{565} = \frac{23 \cdot 6 - 5^2}{5} = 5 - i,$$

$$\rho = 113 - (23 + 6i)(5 - i) = 113 - (121 + 7i) = -8 - 7i.$$

Then $8^2 + 7^2 = 64 + 49 = 113$.