

Name KEY
 (print)

Total = 100 points

Please show all your work.

1. (25 pts) One in four adults say he/she has no trouble sleeping at night. You randomly select five adults and ask if he/she has no trouble sleeping at night.

a) (15 pts) Construct the binomial probability distribution (for this sample of five adults, where x represents the number of adults that have no trouble sleeping at night).

x	P(x)
0	$(3/4)^5 = .237$
1	$5C_1 (3/4)^4 (1/4) = .396$
2	$5C_2 (3/4)^3 (1/4)^2 = .267$
3	$5C_3 (3/4)^2 (1/4)^3 = .088$
4	$5C_4 (3/4)^1 (1/4)^4 = .015$
5	$(1/4)^5 = .001$

5 pts - shows at least one computation

2 pts per p(x)

3 pts completion

Most did quite well on this page!

b) (5 pts) Find the probability that the number of people (from this group of five randomly selected adults) who say that they have no trouble sleeping is at most 2.

if only $P(x) \leq 2$ 20% $P(0) + P(1) + P(2)$
 $.237 + .396 + .267 = .9$

→ (-2)

if $P(X \geq 2) \rightarrow (-1)$

Answer .9 = 90%

c) (5 pts) Find the mean and standard deviation for this binomial distribution.

$\bar{x} = \frac{1}{4} \times 5 = 1.25$ $\sigma = \sqrt{npq} = \sqrt{5 \cdot \frac{3}{4} \cdot \frac{1}{4}} =$

Mean 1.25

Standard Deviation .968

no deduction if students calculated the ~~long~~ long way ($\sum x p(x)$ etc) but if write a note ($\bar{x} = np; \sigma = \sqrt{npq}$)

2. (15 pts) Use the Standard Normal Distribution Table to find:

a) $P(0.27 < z < 1.24) = P(z < 1.24) - P(z < 0.27) = 0.8925 - 0.6064 = 0.2861$

Answer 0.2861 or 28.61%

b) $P(z < 1.5 \text{ or } z > 1.74) = P(z < 1.5) + (1 - P(z < 1.74)) = 0.9332 + (1 - 0.9591) = 0.9741$

if 0.9332 - 0.0409 → (-1)

Answer 0.9741 or 97.41%

3. (25 pts) A study found that the mean height of men (ages 20-29) is 69.9 inches with a standard deviation of 3.0 inches. Assume that the heights are normally distributed.

a) (10 pts) Find the probability that the height of a randomly selected man (age 20-29) is between 64 and 71 inches?

$P(64 \leq x \leq 71)$ $z_{64} = \frac{64 - 69.9}{3.0} = -1.97$
 $= P(-1.97 \leq z \leq 0.37)$ $z_{71} = \frac{71 - 69.9}{3.0} = 0.37$
 $= P(z \leq 0.37) - P(z \leq -1.97)$
 $= 0.6443 - 0.0244 = 0.6199$

Answer 0.6199 or 61.99%

b) (7 pts) If you randomly select 300 men (ages 20-29), about how many of them will be over 76 inches tall?

$P(x > 76) = P(z > 2.03) = 1 - 0.9788 = 0.0212$ $0.0212 \times 300 = 6.36 \sim 6$
 $z = \frac{76 - 69.9}{3} = 2.03$ *if 0.9788 * 300 → (-1)*

Answer about 6

c) (8 pts) Find the height corresponding to the 80th percentile.

$z = 0.84$ $x = 69.9 + 0.84 \times 3.0 = 72.42$

if $x = 69.9 + 80 \cdot 3 \rightarrow (-2)$

72.4 inches

*if 0.0212 → (-2)
if got to z=2.03 only → (-3)*

Answer 72.4 inches

4. (20 pts) The average number of cans of soda per day that each student drinks is given below. Construct a probability distribution and then find the mean and the standard deviation of the probability distribution.

Cups	Students	$P(x)$	$x - \bar{x}$	$(x - \bar{x})^2$
0	21	$21/80 = .26$	-1.67	2.8
1	16	$16/80 = .20$	-.67	.4
2	20	$20/80 = .25$.33	.1
3	15	$15/80 = .19$	1.33	1.8
4	8	$8/80 = .10$	2.33	5.4
	<u>80</u>			

$$\bar{x} = 0 \times .26 + 1 \times .20 + 2 \times .25 + 3 \times .19 + 4 \times .10 = 1.67$$

$$\sigma = \sqrt{2.8 \times .26 + .4 \times .20 + .1 \times .25 + 1.8 \times .19 + 5.4 \times .10} = \sqrt{1.715} \approx 1.3$$

Mean 1.67

Standard deviation 1.3

5. (15 pts) 1,500 raffle tickets are sold at \$2 each for 4 prizes valued at \$800, \$600, \$500, and \$300. You buy one ticket. What is the expected value of your gain/loss?

$$798 \times \frac{1}{1500} + 598 \times \frac{1}{1500} + 498 \times \frac{1}{1500} + 298 \times \frac{1}{1500} + (-2) \times \frac{1496}{1500} = -.53$$

if didn't subtract \$ winning by 2\$. $\rightarrow (-2)$

Answer -0.53

= loss of 53¢