Problem 3. Three applications for the Newton cooling equation $y' = -h(y - y_1)$ are considered, where h, y_1 are constants.

(a) Cooling. An apple initially 23 degrees Celsius is placed in a refrigerator at 2 degrees Celsius. The exponential model is the apple temperature $u(t) = 2 + 21e^{-ht}$. Display the differential equation and the initial condition.

(b) Heating. A beef roast initially 8 degrees Celsius is placed in an oven at 190 degrees Celsius. The exponential model is the roast temperature $u(t) = 190 - 182e^{-ht}$. Display the differential equation and the initial condition.

(c) Fish Length. K. L. von Bertalanffy in 1934 modeled the growth of fish using the equation $\frac{dL}{dt} = h(L_\infty - L(t))$. The fish has mature length L_∞ inches, length $L(t)$ while growing, t is in months and h is the growth rate. Given growth data of $L(0) = 0$, $L(1) = 5$, $L(2) = 7$, find the mature length L_∞, the growth rate h and the months to grow to 95% of mature length.

References. Edwards-Penney section 1.5.
Course notes on **Newton's linear drag model**:
`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/2250kinetics.pdf`
Course notes on **Newton cooling**:
`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/linearapplications2-5.pdf`
Wikipedia biography of **Ludwig von Bertalanffy**:
`http://en.wikipedia.org/wiki/Ludwig_von_Bertalanffy`
Pisces Conservation Ltd **Growth Models**, especially Gompertz, logistic and von Bertalanffy.
`http://www.pisces-conservation.com/growthhelp/index.html`
Serway and Vuille, *College Physics 9/E*, Brooks-Cole (2011), ISBN-10: 0840062060.
The **Coffee Cooling Problem**, a Wolfram Demonstration by S.M. Binder.
`http://demonstrations.wolfram.com/TheCoffeeCoolingProblem/`

Problem 4. Logistic growth $F(x) = rx(1 - x/M)$ can be used to describe the annual natural growth of a fish stock. Symbol $x(t)$ is the stock biomass in number of fish at the start of month t. The intrinsic growth rate is symbol r. The environmental carrying capacity in stock biomass terms is symbol M.

(a) Assume a pond has carrying capacity $M = 780$ thousand fish. If 92% of the the fish survive to maturity, then $r = 0.92$. Display the no-harvesting model $x'(t) = F(x(t))$, using only symbols x and t.

(b) Assume constant harvesting $H \geq 0$ to give the model $x'(t) = F(x(t)) - H$. Use the college algebra quadratic formula to find the equilibrium points in terms of symbols r, M, H. Then verify facts **A, B, C** from your answer.

b-1. If $H = \frac{rM}{4}$, then there is one equilibrium point $x = \frac{M}{2}$ (a double real root).

b-2. If $H > \frac{rM}{4}$, then there is no equilibrium point.

b-3. If $0 < H < \frac{rM}{4}$, then there are two equilibrium points.

(c) Replace symbols r, M by 0.92 and 780. Create a short filmstrip of 5 hand-drawn phase diagrams for the equation $x'(t) = F(x(t)) - H$ using the successive harvest values

$$H = 0, \frac{1}{4}\left(\frac{rM}{4}\right), \frac{1}{2}\left(\frac{rM}{4}\right), \frac{1}{1}\left(\frac{rM}{4}\right), \frac{11}{10}\left(\frac{rM}{4}\right).$$

Each phase diagram shows the equilibria and at least 5 threaded solutions, with labels for funnel, spout and node. The graph window is $t = 0$ to 36 months and $x = 0$ to $2M$.

(d) Justify a guess for the **maximum sustainable harvest**, based on your 5 diagrams. This is an approximate value for the largest catch H that can be taken over 36 months.

References. Edwards-Penney sections 2.1, 2.2.

Course document on the **Logistic Equation**:

`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/2250logistic.pdf`

Course document on **Stability**:

`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/2250phaseline.pdf`

Course document on **Fish Farming**:

. `http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/fishFarming2014.pdf`.

A logistic fish farming investigation in Malaysia by **M.F. Laham 2012**:

`http://www.ukm.my/jsm/pdf_files/SM-PDF-41-2-2012/04%20Mohamed%20Faris.pdf`