Week 2 Examples

Example 1: Classify as separable (S), quadrature (Q), linear (L) or none (N). (1) \( y' = 3(xy)^{1/3} \), (2) \( y' = xy^2 + 1 \), (3) \( y' = \sin(y) \), (4) \( y' = y \sin(x) \), (5) \( y' = e^{\ln|x|} \), (6) \( y' + xy = x^2y \)
Answers: (1) S; (2) N; (3) S; (4) S,L; (5) Q,S,L; (6) L.

Example 2: Check explicit answer \( y = (x^{3/2} + c)^2 \) for \( y' = 3\sqrt{x}\sqrt{y} \) on domain \( x \geq 0, y \geq 0 \).

Example 3: Check implicit answer \( \csc(y) \cot(y) = -x^2/c + c \) for \( y' = x \sin(y) \).

Example 4: Let \( f(x,y) = 1 - x^2 + y^2 - x^2y^2 \). In relation \( f(x,0) = F(x)G(y) \), equations \( f(0,0) = F(0)G(y) \) can determine \( F,G \). Explain. Then find one pair \( F,G \).

Example 5: Solve using the constant equation shortcut or the quadrature shortcut. (1) \( y' + 2y = 6 \), (2) \( 2y' + 5y = 3 \), (3) \( 2y' = 3 \), (4) \( 3y' = 5y + \pi \).

Example 6: Solve using the integrating factor shortcut for homogeneous equations. (1) \( y' + 8xy = 0 \), (2) \( 2y' + \sin(x)y = 0 \), (3) \( xy' + \ln|x|y = 0 \).

Example 7: Solve a non-separable equation using the integrating factor method. (1) \( xy' + 2y = x^2 \), (2) \( xy' + 2y = x \), (3) \( xy' + 2y \ln|x| = \ln|x|e^{(\ln|x|)^2} \).
Answers: (1) \( y = x^2/4 + c/x^2 \), (2) \( y = x/3 + c/x^2 \), (3) \( y = \frac{1}{4} e^{(\ln|x|)^2} + c/e^{(\ln|x|)^2} \).

Example 8: Solve the brine tank model \( \frac{dx}{dt} = 1/4 - x/16 \), \( x(0) = 20 \).

Example 9: Solve the brine tank cascade \( x' = -x/2 \), \( y' = x/2 - y/4 \), \( z' = y/4 - z/6 \) with \( x(0) = 1, y(0) = -2, z(0) = 1.5 \). Answer: \( x = e^{-t/2} \), \( y = -2e^{-t/2} \), \( z = 1.5e^{-t/2} \)

Example 10: Find all equilibrium solutions for \( (x^2 + 1)y' = x + 1 - xy^2 - y^2 \)

Example 11: Solve \( y' = (1 - y)y \) by the substitution \( u = y/(1 - y) \).

Example 12: Solve \( y' = (1 - y)y \) by partial fraction methods.

Example 13: Solve \( y' = 7y(y - 13) \), \( y(0) = 17 \). See 2.1-8.

Example 14: Draw a phase line diagram for \( y' = y(1 - y^2)(y + 1) \).

Example 15: Draw a phase diagram for \( y' = y^2(y^2 - 4) \). See 2.2-17.