Solve by elimination or verify no solution. Give the answer in parametric form.

3.1 #8 P149

$$\begin{cases} 3x - 6y = 12 \\ 2x - 4y = 8 \end{cases}$$

- Please document steps as combo, mult, swap
- Identify reduced echelon sys.
- Check the answer.

$$\begin{cases} x - 2y = 4 \\ 2x - 4y = 8 \end{cases}$$

$$\begin{cases} x - 2y = 4 \\ \quad\;\; 0 = 0 \end{cases}$$

$$\begin{cases} x = 4 + 2t, \\ y = t, \quad -\infty < t < \infty. \end{cases}$$

lead variable x

free variable y

3.1 #14 P149

$$\begin{cases} 4x + 9y + 12z = -1, \\ 3x + y + 16z = -46, \\ 2x + 7y + 3z = 19. \end{cases}$$

also good documentation ↓ Best Documentation ↓

$$\begin{cases} x + 8y - 4z = 45, \\ 3x + y + 16z = -46 \\ 2x + 7y + 3z = 19 \end{cases}$$

(#1) = (#1) − (#2) Combo
Copy #2
Copy #3

$$\begin{cases} x + 8y - 4z = 45, \\ 0 - 23y + 28z = -181, \\ 0 - 9y + 11z = -71. \end{cases}$$

copy #1
(#2) = (#2) − 3(#1) Combo
(#3) = (#3) − 2(#1) Combo

$$\begin{cases} x + 8y - 4z = 45, \\ 0 - 5y + 6z = -39, \\ 0 - 9y + 11z = -71. \end{cases}$$

Copy #1
(#2) = (#2) − 2(#3) Combo
Copy #3

$$\begin{cases} x + 8y - 4z = 45, \\ 0 - 5y + 6z = -39, \\ 0 + y - z = 7. \end{cases}$$

Copy #1
Copy #2
(#3) = (#3) − 2(#2) Combo

3.1 #14 P149 cont.

$$\begin{cases} x + 8y - 4z = 45, \\ 0 + 0 + z = -4 \\ 0 + y - z = 7. \end{cases}$$

Copy #1
(#2) = (#2) + 5(#3) Combo
Copy #3

$$\begin{cases} x + 8y - 4z = 45, \\ 0 + y - z = 7, \\ 0 + 0 + z = -4. \end{cases}$$

Swap #2, #3 Swap

$$\begin{cases} x + 8y + 0 = 29, \\ 0 + y + 0 = 3, \\ 0 + 0 + z = -4 \end{cases}$$

(#1) = (#1) + 4(#3) Combo
(#2) = (#2) + (#3)

$$\begin{cases} x + 0 + 0 = 5, \\ 0 + y + 0 = 3, \\ 0 + 0 + z = -4. \end{cases}$$

(#1) = (#1) − 8(#2)

$$\begin{cases} x = 5, \\ y = 3, \\ z = -4 \end{cases}$$

final solution

3.1 #30 P149 Given general solution $y = Ae^{4x/3} + Be^{-7x/5}$ for $15y'' + y' - 28y = 0$, solve for A, B to satisfy initial conditions $y(0) = 41$, $y'(0) = 164$

The answer is $\boxed{A = 81,\ B = -40}$. Justified below.

$$\begin{cases} y(0) = 41 \\ y'(0) = 164 \end{cases}$$

Given initial data, $y = Ae^{4x/3} + Be^{-7x/5}$

$$\begin{cases} Ae^0 + Be^0 = 41 \\ \frac{4A}{3}e^0 + \left(\frac{-7B}{5}\right)e^0 = 164 \end{cases}$$

substitute for y, set $x = 0$.

3.1 #30 p149
cont.

$$\begin{cases} A + B = 41 \\ 20A - 21B = 2460 \end{cases}$$ use $e^0 = 1$ and clear fractions

$$\begin{cases} A + B = 41 \\ 0 - 41B = 1640 \end{cases}$$ To #2 add $-20(\#1)$

$$\begin{cases} A + B = 41 \\ 0 + B = -40 \end{cases}$$ Divide #2 by -41

$$\begin{cases} A + 0 = 81 \\ 0 + B = -40 \end{cases}$$ To #1 add $-(\#2)$

Check: $y(0) = A + B = 81 - 40 = 41$ $\qquad y'(0) = \frac{4A}{3} - \frac{7B}{5} = 4(27) - 7(-8) = 164$

3.2 #9 p160 Solve by back substitution

$$\begin{cases} 2x_1 + x_2 + x_3 + x_4 = 6, \\ 3x_2 - x_3 - 2x_4 = 2, \\ 3x_3 + 4x_4 = 9, \\ x_4 = 6 \end{cases}$$

The three operations on equations are applied to reduce the system to a reduced row-echelon system. This will be done by matrix methods

$$\left(\begin{array}{cccc|c} 2 & 1 & 1 & 1 & 6 \\ 0 & 3 & -1 & -2 & 2 \\ 0 & 0 & 3 & 4 & 9 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right)$$ augmented matrix. Non-diagonal entries must be cleared to zero.

$$\left(\begin{array}{cccc|c} 2 & 1 & 1 & \boxed{0} & 0 \\ 0 & 3 & -1 & \boxed{0} & 14 \\ 0 & 0 & 3 & \boxed{0} & -15 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right)$$ Add multiples of #4 to #1, #2, #3 to create zeros

$$\left(\begin{array}{cccc|c} 2 & 1 & 1 & 0 & 0 \\ 0 & 3 & -1 & 0 & 14 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right)$$ Divide #3 by 3. Non-diagonal entries are yet to be cleared to zero.

$$\left(\begin{array}{cccc|c} 2 & 1 & \boxed{0} & 0 & 5 \\ 0 & 3 & \boxed{0} & 0 & 9 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right)$$ Add multiples of #3 to #1, #2 to create zeros

$$\left(\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right)$$ Divide #2 by 3

$$\left(\begin{array}{cccc|c} 1 & \boxed{0} & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right)$$ Last zero created for non-diagonal entries. RREF found.

$$\begin{cases} x_1 = 1 \\ x_2 = 3 \\ x_3 = -5 \\ x_4 = 6 \end{cases}$$ Final Solution

3.2 #22 p160 Transform to RREF and solve

$$\begin{cases} 4x_1 - 2x_2 - 3x_3 + x_4 = 3 \\ 2x_1 - 2x_2 - 5x_3 + 0 = -10 \\ 4x_1 + x_2 + 2x_3 + x_4 = 17 \\ 3x_1 + 0 + x_3 + x_4 = 12 \end{cases}$$

answer: $x_1 = 3$, $x_2 = -2$, $x_3 = 4$, $x_4 = -1$

$$\begin{pmatrix} 4 & -2 & -3 & 1 & 3 \\ 2 & -2 & -5 & 0 & -10 \\ 4 & 1 & 2 & 1 & 17 \\ 3 & 0 & 1 & 1 & 12 \end{pmatrix} = A$$

augmented Matrix. will find RREF using The 3 operations, allowing no fractions to form.

$$\begin{pmatrix} 1 & -2 & -4 & 0 & -9 \\ 2 & -2 & -5 & 0 & -10 \\ 4 & 1 & 2 & 1 & 17 \\ 3 & 0 & 1 & 1 & 12 \end{pmatrix} = A_1$$

To #1 add $-(\#4)$. This creates a leading 1. Boxed items should be zeroed.

$$\begin{pmatrix} 1 & -2 & -4 & 0 & -9 \\ 0 & 2 & 3 & 0 & 8 \\ 0 & 9 & 18 & 0 & 53 \\ 0 & 6 & 13 & 1 & 39 \end{pmatrix} = A_4$$

To #2, #3, #4 add multiples of #1 to create zeros.

To implement The Three operations in MAPLE, use The ideas below, which apply to The steps above to get A, A_1, A_4.

```
A := matrix( [[4,-2,-3,1,3],[2,-2,-5,0,-10],
              [4,1,2,1,17],[3,0,1,1,12]]);
A1 := addrow(A,4,1,-1);     A2 := addrow(A1,1,2,-2);
A3 := addrow(A2,1,3,-4);    A4 := addrow(A3,1,4,-3);
A5 := addrow(A4,2,3,-4);    A6 := addrow(A5,2,4,-3);
```

$$\begin{pmatrix} 1 & -2 & -4 & 0 & -9 \\ 0 & 2 & 3 & 0 & 8 \\ 0 & 1 & 6 & 1 & 21 \\ 0 & 0 & 4 & 1 & 15 \end{pmatrix} = A_6$$

perform operations A_5, A_6 above.

$$\begin{pmatrix} 1 & -2 & -4 & 0 & -9 \\ 0 & 0 & -1 & 0 & -4 \\ 0 & 1 & 6 & 1 & 21 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} = A_9$$

use addrow again to create A_7, A_8, A_9.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} = A_{15}$$

similar operations $A_{10} - A_{15}$ find The RREF. see also <u>swaprow</u> and <u>mulrow</u> in Maple.

<u>3.2-26, p160</u> In $\begin{cases} 3x + 2y = 1 \\ 7x + 5y = k \end{cases}$ determine k such That (a) There is a unique solution, (b) no solutions, (c) ∞-many solutions.

The answers:

(a) Unique solution $x = 5 - 2k$, $y = 3k - 7$ for all k

(b) Never happens.

(c) Never happens.

All answers are based upon The calculation (details elsewhere)

$$\text{RREF}\left(\begin{array}{cc|c} 3 & 2 & 1 \\ 7 & 5 & k \end{array}\right) = \left(\begin{array}{cc|c} 1 & 0 & 5-2k \\ 0 & 1 & 3k-7 \end{array}\right).$$

Check: $3x + 2y = 3(5-2k) + 2(3k-7) = 15 - 6k + 6k - 14 = 1$ ✓

$7x + 5y = 7(5-2k) + 5(3k-7) = 35 - 14k + 15k - 35 = k$ ✓

<u>3.3-14 p169</u> Given $A = \begin{bmatrix} 1 & 3 & 2 & 5 \\ 2 & 5 & 2 & 3 \\ 2 & 7 & 7 & 22 \end{bmatrix}$, find rref(A).

This problem is best worked with maple assist, using

```
with(linalg):
A:= matrix([[1,3,3,5],[2,5,2,3],[2,7,7,22]]);
A1:= addrow(A,1,2,-2);
A2:= addrow(A1,1,3,-2);
   ⋮
A6:= addrow(A5,3,1,-2);
A7:= addrow(A6,2,1,-3);
```

$$A_7 = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

<u>Ans Check</u>: The last matrix implies $x = 4, y = -3, z = 5$ so

$$x + 3y + 2z = 4 - 9 + 10$$

3.3-14 P 169 Given $A=\begin{bmatrix}1&3&2&5\\2&5&2&3\\2&7&7&22\end{bmatrix}$, find rref(A).

This problem is best worked with maple assist, using

```
with(linalg):
A:= matrix([[1,3,2,5],[2,5,2,3],[2,7,7,22]]);
A1:= addrow(A,1,2,-2);
A2:= addrow(A1,1,3,-2);
  :
A6:= addrow(A6,3,1,-2);
A7:= addrow(A6,2,1,-3);
```

$A_7=\begin{bmatrix}1&0&0&4\\0&1&0&-3\\0&0&1&5\end{bmatrix}$

Ans Check: The last matrix implies $x=4, y=-3, z=5$

So

$$x+3y+2z = 4-9+10 = 5 \checkmark$$

The other two equations check, also.

3.3-20, P169 Given $A=\begin{bmatrix}3&6&1&7&13\\5&10&8&18&47\\2&4&5&9&26\end{bmatrix}$, find rref(A).

See 3.3-14 for Maple assist. Some commands:

```
A1:= addrow(A,3,2,-2);
A2:= swaprow(A1,1,2);
  :
A5:= mulrow(A4,2,1/7);
  :
A10 is the rref
rref(A);   checks the answer.
```

3.4-20, P 182 Write as $Ax=0$ and solve for x in vector form:

$$\begin{cases} x_1-3x_2+7x_5=0\\ x_3-2x_5=0\\ x_4-10x_5=0\end{cases}$$

$$\begin{pmatrix}1&-3&0&0&7\\0&0&1&0&-2\\0&0&0&1&-10\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

vector form $Ax=0$. already rr

$$\left\{\begin{aligned}x_1&=\\x_2&=\\x_3&=\\x_4&=\\x_5&=\end{aligned}\right.$$

Lead vars: x_1, x_3, x_4

free vars: x_2, x_5

$$\begin{pmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{pmatrix}= s\begin{pmatrix}\\ \\ \\ \end{pmatrix}+t\begin{pmatrix}\\ \\ \\ \end{pmatrix}$$

vector form of Solution x $-\infty<s,t<\infty$.

Check:

$$A\begin{pmatrix}\\ \\ \end{pmatrix}=\begin{pmatrix}\\ \\ \end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$A\begin{pmatrix}\\ \\ \end{pmatrix}=\begin{pmatrix}\\ \\ \end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

Scheme is to Test The basis elements. If They satisfy The eq $A\vec{x}=\vec{0}$ Then so does The cand. date sol.

3.5-22, P 195 Find A^{-1} given $A = \begin{bmatrix} 4 & 0 & 1 & 1 \\ 3 & 1 & 3 & 1 \\ 0 & 1 & 2 & 0 \\ 3 & 2 & 4 & 1 \end{bmatrix}$

$$B = \left[\begin{array}{cccc|cccc} 4&0&1&1&1&0&0&0 \\ 3&1&3&1&0&1&0&0 \\ 0&1&2&0&0&0&1&0 \\ 3&2&4&1&0&0&0&1 \end{array}\right]$$

B = aug(A, I)
Augment I to A.

Find rref(B) using maple assist operations

```
swaprow(B,n,m);
mulrow(B,k,x);
addrow(B,source,target,x);
```

Then $\text{rref}(B) = [\, I \mid A^{-1} \,]$. Check the answer by multiplication of AA^{-1} to get I. Also, maple command `inverse(A);` gives the inverse [not a valid solution, but a valid answer check].

3.5-28, P 196 Solve for X in $AX = B$ given $A = \begin{bmatrix} 6 & 5 & 3 \\ 5 & 3 & 2 \\ 3 & 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 0 & 2 \\ -1 & 3 & 5 & 0 \\ 1 & 1 & 0 & 5 \end{bmatrix}$.

The answer is $X = A^{-1}B$, found from $AX = B$ by multiplying across by A^{-1}. To find A^{-1}, apply the ideas of 3.5-22 above. A second method is to form

$$C = \begin{bmatrix} 6 & 5 & 3 & 2 & 1 & 0 & 2 \\ 5 & 3 & 2 & -1 & 3 & 5 & 0 \\ 3 & 4 & 2 & 1 & 1 & 0 & 5 \end{bmatrix}$$

and find rref(C). The answer $X = A^{-1}B$ will then appear as the last 4 columns of C. Hand solutions use a maple assist and maple check as outlined in 3.5-22 above.

3.6-3, P 213 Evaluate $\begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 5 & 0 \\ 3 & 6 & 9 & 8 \\ 4 & 0 & 10 & 7 \end{vmatrix}$ by cofactor expansion.

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 5 & 0 \\ 3 & 6 & 9 & 8 \\ 4 & 0 & 10 & 7 \end{vmatrix} = (1)\begin{vmatrix} 0 & 5 & 0 \\ 6 & 9 & 8 \\ 0 & 10 & 7 \end{vmatrix} + 0(\text{3 other 3x3 determinants})$$

[Cofactor expansion along row 1]

$$= (1)\left[0\begin{vmatrix} \cdot & \cdot \\ \cdot & \cdot \end{vmatrix} + (-1)(6)\begin{vmatrix} 5 & 0 \\ 10 & 7 \end{vmatrix} + 0\begin{vmatrix} \cdot & \cdot \\ \cdot & \cdot \end{vmatrix} \right]$$

[Cofactor expansion along col 1]

$= (1)(-1)(6)(35)$, by Sarrus' Rule

$= -210$

ans checks /w A-34

3.6-19, P 214 Evaluate by the method of elimination.

$$\begin{vmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & -2 & 0 \\ -2 & 3 & -2 & 3 \\ 0 & -3 & 3 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 3 & -2 & 9 \\ 0 & -3 & 3 & 3 \end{vmatrix}$$ Add to row 3: 2 times row 1

$$= \begin{vmatrix} 1 & -2 & 0 \\ 3 & -2 & 9 \\ -3 & 3 & 3 \end{vmatrix}$$ By cofactor expansion along col 1.

$$= \begin{vmatrix} 1 & -2 & 0 \\ 0 & 4 & 9 \\ 0 & -3 & 3 \end{vmatrix}$$ Add multiples of row 1 to rows 2, 3.

$$= \begin{vmatrix} 4 & 9 \\ -3 & 3 \end{vmatrix}$$ By cofactor expansion along column 1.

$= 12 + 27$ Sarrus' Rule

$= \boxed{39}$ Answer checks /w A-34

3.6-30, p 214 Solve by Cramer's Rule

$$\begin{cases} x_1 + 4x_2 + 2x_3 = 3 \\ 4x_1 + 2x_2 + x_3 = 1 \\ 2x_1 - 2x_2 - 5x_3 = 3 \end{cases}$$

This is different from #29, #30, #31!

answer: $x_1 = -1/7$, $x_2 = 39/28$, $x_3 = -17/14$

$$\Delta = \begin{vmatrix} 1 & 4 & 2 \\ 4 & 2 & 1 \\ 2 & -2 & 5 \end{vmatrix} = 56 \qquad \Delta_1 = \begin{vmatrix} 3 & 4 & 2 \\ 1 & 2 & 1 \\ 3 & -2 & -5 \end{vmatrix} = -8 \qquad \Delta_2 = \begin{vmatrix} 1 & 3 & 2 \\ 4 & 1 & 1 \\ 2 & 3 & -5 \end{vmatrix} = 78$$

$$\Delta_3 = \begin{vmatrix} 1 & 4 & 3 \\ 4 & 2 & 1 \\ 2 & -2 & 3 \end{vmatrix} = -68$$

$$x_1 = \frac{\Delta_1}{\Delta} = -\frac{1}{7} \qquad x_2 = \frac{\Delta_2}{\Delta} = \frac{39}{28} \qquad x_3 = \frac{\Delta_3}{\Delta} = \frac{-17}{14}$$

Check: $x_1 + 4x_2 + 2x_3 = -\frac{1}{7} + \frac{39}{7} + \frac{-17}{7} = 3$ ok

$4x_1 + 2x_2 + x_3 = -\frac{4}{7} + \frac{39}{14} - \frac{17}{14} = 1$ ok

$2x_1 - 2x_2 - 5x_3 = \frac{-2}{7} - \frac{39}{14} + \frac{5(17)}{14} = 3$ ok

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3.6-38, p 214 Find the inverse A^{-1} by using the adjoint formula $A^{-1} = \frac{\text{adj} A}{\det A}$, for

$$A = \begin{bmatrix} 3 & 4 & -3 \\ 3 & 2 & -1 \\ -3 & 2 & -4 \end{bmatrix}.$$

$$\text{adj}(A) = \begin{bmatrix} +(-6) & -(-15) & +(12) \\ -(-10) & +(-21) & -(18) \\ +(2) & -(6) & +(-6) \end{bmatrix}^T$$

minors in parens, checkerboard signs.

$$= \begin{bmatrix} -6 & 10 & 2 \\ 15 & -21 & -6 \\ 12 & -18 & -6 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 3 & 4 & -3 \\ 3 & 2 & -1 \\ -3 & 2 & -4 \end{vmatrix} = 3(-6) - 3(-10) - 3(2) = 6$$

cofactor exp.

$$A^{-1} = \frac{\text{adj} A}{\det A} = \frac{1}{6}\begin{bmatrix} -6 & 10 & 2 \\ 15 & -21 & -6 \\ 12 & -18 & -6 \end{bmatrix}$$

check: $$A A^{-1} = \frac{1}{6}\begin{bmatrix} 3 & 4 & -3 \\ 3 & 2 & -1 \\ -3 & 2 & -4 \end{bmatrix}\begin{bmatrix} -6 & 10 & 2 \\ 15 & -21 & -6 \\ 12 & -18 & -6 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} -18+60-36 & 0 & 0 \\ 0 & 30-42+18 & 0 \\ 0 & 0 & -6+2+24 \end{bmatrix}$$

$$= I.$$