

**Math 2280 Numerical Methods Homework  
S2019**

**References:** Edwards-Penney, Sections 2.4,2.5, 2.6. This document is located at:

<http://www.math.utah.edu/~gustafso/s2019/2280/homework/numericalDEproject/numericalDEproject-S2019.pdf>

Other related and required documents:

**2280 web site:** <http://www.math.utah.edu/~gustafso/s2019/2280/>

**Slides:** <http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/numericalDE2008.pdf>

**Manuscript:** <http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/numericalManuscript.pdf>

**Maple code:** <http://www.math.utah.edu/~gustafso/s2019/2280/homework/numericalDEproject/src/2280numerical-hints.txt>.

### Symbolic Solution. (E & P Exercises 2.4-6, 2.5-6, 2.6-6 Symbolic Solution)

The symbolic solution of  $y' = -2xy$ ,  $y(0) = 2$  is  $y = 2e^{-x^2}$ . Display the details for the derivation of this symbolic solution, using methods from Edwards-Penney section 1.4 or 1.5. Do a full 2-panel answer check.

The answer  $y = 2e^{-x^2}$  can be used to make the following table, which is used in the problems below.

$x$	0.00	0.10	0.20	0.25	0.30	0.40	0.50
$2e^{-x^2}$	2.000000000	1.980099667	1.921578878	1.878826126	1.827862371	1.704287578	1.557601566

### Euler's Method. (E & P Exercise 2.4-6)

Consider the initial value problem  $y' = -2xy$ ,  $y(0) = 2$  with symbolic solution  $y = 2e^{-x^2}$ . Apply Euler's method to produce two dot tables, as shown below. The first has three pairs,  $h = 0.25$ . The second has six pairs,  $h = 0.1$ . Reproduce the summary of results below, writing 4 small digits (Geek pen required) into the blank rectangle . The 4th digit can be rounded or not.

The work for  $h = 0.25$  is to be entirely hand-written, with calculator assist. Answer checks and the work for  $h = 0.1$  may use technology.

Table  $h = 0.25$ :  $[0, 2]$ ,  $[0.25, 2]$ ,  $[0.5, 1.75$  ]

Table  $h = 0.10$ :  $[0, 2]$ ,  $[0.1, 2.0]$ ,  $[0.2, 1.96]$ ,  $[0.3, 1.88$  ],  $[0.4, 1.76$  ],  $[0.5, 1.627$  ]

Actual $y(.5)$	Approx $y(.5)$ , $h = 0.25$	Approx $y(.5)$ , $h = 0.10$
1.557601566	1.75 <input type="text"/>	1.627 <input type="text"/>

### Heun's Method (Modified Euler). (E & P Exercise 2.5-6)

Consider the initial value problem  $y' = -2xy$ ,  $y(0) = 2$  with symbolic solution  $y = 2e^{-x^2}$ . Apply Heun's method (Improved Euler) to produce one dot table of six rows,  $h = 0.1$ . Reproduce the table below and fill in missing digits. Hand-written work is expected with computer assist. Hand-written work with calculator assist should end after the estimate for  $y(0.1)$ . Technology should be used for the remaining answers. Four digits are expected in .

$x$	0.0	0.1	0.2	0.3	0.4	0.5
Actual $y(x)$	2.000000000	1.980099667	1.921578878	1.827862371	1.704287578	1.557601566
Approx $y(x)$	2.0000000	1.980 <input type="text"/>	1.921 <input type="text"/>	1.827 <input type="text"/>	1.704 <input type="text"/>	1.557 <input type="text"/>

### RK4 Method. (E & P Exercise 2.6-6)

Consider the initial value problem  $y' = -2xy$ ,  $y(0) = 2$  with symbolic solution  $y = 2e^{-x^2}$ . Apply the RK4 method to produce one dot table of three rows,  $h = 0.25$ . Reproduce the table below, filling in the missing digits. Hand-written work should complete the estimate of  $y(0.25)$  to 8 digits, using calculator assist. The remaining steps should use technology, with a plan to reproduce the hand-written result. Four digits are expected in .

$x$	0.00	0.25	0.50
Actual $y(x)$	2.000000000	1.878826126	1.557601566
Approx $y(x)$	2.000000000	1.878 <input type="text"/>	1.557 <input type="text"/>

**Submit this printed page**, with blanks filled in, as the first page of your report. Attach hand-written solutions next. Append computer results last, then staple. Assigned in Week 3.