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## Productive Economy


#### Abstract

In order to figure out the relationship between economic input and output in the United States in 2008, 2008 Summary Use Annual I-O Table was used to judge whether the economy was productive or not by establishing a consumption matrix.


## Input and output information

2008 Summary Use Annual I-O Table, namely, the Input and output information were obtained from network database (http://www.bea.gov/industry/io\ annual.htm ) . The obtained table was in CSV format, so I sorted the US Economy Use Data 2008 into a matrix A of $65 * 65$ and the Total Output Vector into a diagonal matrix P of $65 * 65$ in EXCEL. Matrixes as follows:
$\mathrm{U}=[[49079,202,0,0,0,1,1445,0,0,0,0,0,0,0,0,0,0,39,194737,2072,0,0,0,0,1407,0,318,988,0,0,0$, $0,0,0,7,0,1,0,0,0,0,0,0,0,129,0,0,0,549,28,100,0,167,1,33,0,7,67,163,2899,21,-21,8,2351,0]$ [17893,6913,0,0,0,0,0,15212,5,0,0,0,0,10,9,0,60,247,6853, $0,41,3096,86,3,583,2181,0,781,0,0$ , $2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,157,8,1,0,63,0,113,21,3,105,454,4533,54,0,19,1369,0]$ [0,0,32679,5,7,122936,0,23,7,38,10,14,0,2,9,4,4,3,26,6,0,23,5,506245,8230,20,425,293,0,0,0, $1,0,3305,0,21,21,8,372,9,0,20,12,0,35,168,24,8,641,175,130,34,23,101,179,60,29,57,270,403$, $168,0,460,0,9394]$
[51,6,0,24,2,148,45,0,0,24,55,382,17,137,0,0,0,25,0,5,0,0,175,51,45,0,10445,7463,0,1,373,69 $49,32,137,2196,429,2374,314,704,831,4963,1721,163,0,66,1030,537,303,4014,245,1349,286$, 820,2784,3228,221,185,123,2782,5019,2358,230,52,4059,46]
$[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0$ ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
$[0,2,0,0,0,9,0,36,54,252,113,61,43,18,69,43,51,39,342,25,11,109,36,16,254,109,280,475,0,0$, $0,36,0,0,13,49,80,16,588,37,221,58,17,0,803,558,260,95,740,87,460,228,51,994,290,1148,41$, 84,511,542,548,619,274,3217,2439]]
$\mathrm{P}=[[, 329199,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0$, 0,0,0,0,0,0, $0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$ $[0,55085,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0$, $0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$
$[0,0,299490,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0$, 0, $0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$
...
...
...
$[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0$ , $0,0,0,0,0,0,0,0,0,0,0,0,77757,0,0]$
$[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0$ , $0,0,0,0,0,0,0,0,0,0,0,0,0,1452431,0]$
$[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0$ $, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,75620]]$

## Solution

Consumption matrix A tells how much of each input goes into a unit of output. It is calculated by the ratio of Use Matrix $U$ and Total Industry Output Vector, namely, $A=U / P$ :
$A=\left[\left[0.149,0.00367,0 ., 0 ., 0 ., 1.86^{*} 10^{\wedge}(-\right.\right.$
6),0.00107,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.000262,0.261,0.0394,0.,0.,0.,0.,0.00209,0.,0.000255,0. $000854,0 ., 0 ., 0 ., 0 ., 0 ., 0 ., 0.0000474,0 ., 4.32 * 10 \wedge(-$
$6), 0 ., 0 ., 0 ., 0 ., 0 ., 0 ., 0 ., 0.0000541,0 ., 0 ., 0 ., 0.000439,0.0000719,0.000167,0 ., 0.000570,1.26^{*} 10^{\wedge}(-$ 6),0.0000384,0.,0.0000627,0.000431,0.00108,0.00459,0.0000288,$0.0000225,0.000103,0.00162,0$.$] ,$ [0.0544,0.125,0.,0.,0.,0.,0.,0.160,0.0000479,0.,0.,0.,0.,0.0000867,0.0000229,0.,0.000812,0.00 $166,0.00918,0 ., 0.00129,0.0190,0.00114,4.10^{*} 10^{\wedge}(-$
6),0.000868,0.0108,0.,0.000675,0.,0.,0.0000510,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0. $000126,0.0000205,1.67 * 10^{\wedge}(-$ 6), $0 ., 0.000215,0 ., 0.000132,0.000155,0.0000269,0.000676,0.00300,0.00717,0.0000740,0 ., 0.0$ 00244,0.000943,0.], [0.,0.,0.109,0.0000535,0.0000400,0.228,0.,0.000242,0.0000671,0.000151,0.0000305,0.00004 00,0.,0.0000173,0.0000229,0.0000151,0.0000541,0.0000201,0.0000348,0.000114, $0 ., 0.00014$

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1,0.0000661,0.692,0.0123,0.0000988,0.000340,0.000253,0.,0.,0.,3.36*10^(-
6),0.,0.118,0.,0.000353,0.0000908,0.0000788,0.000701,0.0000628,0.,0.0000472,0.0000175,0
.,0.0000147,0.000427,0.0000836,0.0000211,0.000513,0.000449,0.000217,0.000385,0.000078
5,0.000127,0.000209,0.000442,0.000260,0.000367,0.00179,0.000638,0.000230,0.,0.00592,0.,
0.124],
...
...
[0.000155,0.000109,0.,0.000257,0.0000114,0.000275,0.0000333,0.,0.,0.0000954,0.000168,0.
00109,0.0000454,0.00119,0.,0.,0.,0.000168,0.,0.0000950,0.,0.,0.00231,0.0000697,0.0000670,
0.,0.00836,0.00645,0.,0.0000135,0.00952,0.0234,0.000696,0.00490,0.0149,0.00721,0.0103,0.
00309,0.00133,0.00580,0.00578,0.00406,0.000237,0.,0.0000277,0.00262,0.00187,0.000798,0
.00321,0.000629,0.00225,0.00324,0.00280,0.00351,0.00376,0.00163,0.00166,0.000792,0.018
4,0.00794,0.00323,0.000246,0.000669,0.00279,0.000608],
[0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,
0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.],
[0.,0.0000363,0.,0.,0.,0.0000167,0.,0.000379,0.000518,0.00100,0.000345,0.000174,0.000115
,0.000156,0.000176,0.000163,0.000690,0.000262,0.000458,0.000475,0.000346,0.000668,0.0
00476,0.0000219,0.000378,0.000539,0.000224,0.000411,0.,0.,0.,0.000121,0.,0.,0.0000880,0.
000823,0.000346,0.000158,0.00111,0.000258,0.000257,0.000137,0.0000247,0.,0.000337,0.0
0142,0.000906,0.000250,0.000592,0.000223,0.000767,0.00258,0.000174,0.00125,0.000338,0
.00845,0.000367,0.000541,0.00338,0.000858,0.000751,0.000662,0.00352,0.00221,0.0323]]
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In the individual economy, some of the output production was consumed in the production process, while others were transported to meet external demand. We defined the external demand as the matrix Y , and its relationship with the output vector P and the consumption matrix $A$ was: $p-A^{*} p=y$ or $p=(I-A)^{-1} y$. $I 65$ (I cannot be used as the matrix name in the software) is $65^{*} 65$.So the determinant of $(I-A)$ was not 0 , which meant that I-A was invertible. So I evaluated $(\mathrm{I}-\mathrm{A})^{-1}$, the results were follows: $(\mathrm{I}-\mathrm{A})^{-1}=[[1.20,0.728 * 10(-2), 0.506 * 10(-3), 0.456 * 10(-3), 0.105 * 10(-2), 0.380 * 10(-3)$, $0.234 * 10(-2), 0.282 * 10(-2), 0.159 * 10(-2), 0.772 * 10(-3), 0.881 * 10(-3), 0.975 * 10(-3)$, $0.714 * 10(-3), 0.855 * 10(-3), 0.188 * 10(-2), 0.995 * 10(-3), 0.324 * 10(-2), 0.190 * 10(-2), .375)$, $0.612 * 10(-1), 0.210 * 10(-1), 0.377 * 10(-2), 0.378 * 10(-2), 0.476 * 10(-3), 0.536 * 10(-2)$, $0.311 * 10(-2), 0.182 * 10(-2), 0.741 * 10(-2), 0.875 * 10(-3), 0.772 * 10(-3), 0.762 * 10(-3)$, $0.687 * 10(-3), 0.268 * 10(-3), 0.614 * 10(-3), 0.570 * 10(-3), 0.376 * 10(-3), 0.197 * 10(-2)$, $0.842 * 10(-3), 0.934 * 10(-3), 0.111 * 10(-2), 0.163 * 10(-2), 0.515 * 10(-3), 0.237 * 10(-3)$, $0.384 * 10(-3), 0.544 * 10(-3), 0.509 * 10(-3), 0.647 * 10(-3), 0.585 * 10(-3), 0.195 * 10(-2)$, $0.835 * 10(-3), 0.117 * 10(-2), 0.894 * 10(-3), 0.742 * 10(-2), 0.957 * 10(-3), 0.933 * 10(-2)$, $0.820 * 10(-2), 0.707 * 10(-2), 0.628 * 10(-2), 0.108 * 10(-1), 0.380 * 10(-1), 0.112 * 10(-2)$, $\left.0.122^{*} 10(-2), 0.637^{*} 10(-2), 0.116^{*} 10(-1), 0.255^{*} 10(-2)\right]$
,
$[0.768 * 10(-1), 1.15,0.792 * 10(-3), 0.842 * 10(-3), 0.198 * 10(-2), 0.522 * 10(-3), 0.740 * 10(-2),$. 232), $0.305 * 10(-2), 0.158 * 10(-2), 0.131 * 10(-2), 0.221 * 10(-2), 0.107 * 10(-2), 0.221 * 10(-2)$, $0.394 * 10(-2), 0.151 * 10(-2), 0.243 * 10(-1), 0.650 * 10(-2), 0.388^{*} 10(-1), 0.635^{*} 10(-2)$, $0.453 * 10(-2), 0.347 * 10(-1), 0.106 * 10(-1), 0.657 * 10(-3), 0.295 * 10(-2), 0.167 * 10(-1)$,

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0.107*10(-2), 0.234*10(-2), 0.531*10(-3), 0.437*10(-2), 0.538*10(-3), 0.799*10(-3),
0.312*10(-3), 0.970*10(-3), 0.494*10(-3), 0.611*10(-3), 0.295*10(-2), 0.145*10(-2),
0.116*10(-2), 0.238*10(-2), 0.101*10(-2), 0.431*10(-3), 0.243*10(-3), 0.295*10(-3),
0.128*10(-2), 0.125*10(-2), 0.483*10(-3), 0.357*10(-3), 0.906*10(-3), 0.568*10(-3),
0.811*10(-3), 0.844*10(-3), 0.142*10(-2), 0.867*10(-3), 0.185*10(-2), 0.253*10(-2),
0.133*10(-2), 0.184*10(-2), 0.661*10(-2), 0.136*10(-1), 0.866*10(-3), 0.764*10(-3),
0.182*10(-2), 0.411*10(-2), 0.527*10(-2)],
[0.113, 0.324*10(-1), 1.15, 0.610*10(-1), 0.626*10(-1), 0.278, 0.661*10(-1), 0.486*10(-1),
0.660*10(-1), 0.668*10(-1), 0.384*10(-1), 0.362*10(-1), 0.186*10(-1), 0.368*10(-1),
0.377*10(-1), 0.272*10(-1), 0.356*10(-1), 0.277*10(-1), 0.663*10(-1), 0.763*10(-1),
0.401*10(-1), 0.713*10(-1), 0.675*10(-1), 0.838, 0.123, 0.711*10(-1), 0.281*10(-1),
0.211*10(-1), 0.211,0 .144, 0.228,0 .182, 0.933*10(-1), 0.197,0 .135, 0.282*10(-1),
0.350*10(-1), 0.114*10(-1), 0.239*10(-1), 0.185*10(-1), 0.181*10(-1), 0.907*10(-2),
0.413*10(-2), 0.533*10(-2), 0.108*10(-1), 0.133*10(-1), 0.944*10(-2), 0.705*10(-2),
0.158*10(-1), 0.138*10(-1), 0.504*10(-1), 0.416*10(-1), 0.206*10(-1), 0.167*10(-1),
0.191*10(-1), 0.216*10(-1), 0.156*10(-1), 0.155*10(-1), 0.525*10(-1), 0.274*10(-1),
0.138*10(-1), 0.197*10(-1), 0.739*10(-1), 0.422*10(-1), .400] ),
...
...
...
[0.260*10(-2), 0.976*10(-3), 0.110*10(-2), 0.252*10(-2), 0.294*10(-2), 0.110*10(-2),
0.227*10(-2), 0.340*10(-2), 0.368*10(-2), 0.300*10(-2), 0.274*10(-2), 0.394*10(-2),
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0.267*10(-2), 0.365*10(-2), 0.319*10(-2), 0.259*10(-2), 0.299*10(-2),0.233*10(-2),
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$0.313 * 10(-2), 0.319 * 10(-2), 0.280 * 10(-2), 0.263 * 10(-2), 0.618 * 10(-2), 0.132 * 10(-2)$,
$0.205 * 10(-2), 0.236 * 10(-2), 0.102 * 10(-1), 0.820 * 10(-2), 0.244 * 10(-2), 0.270 * 10(-2)$,
$0.122 * 10(-1), 0.268 * 10(-1), 0.144 * 10(-2), 0.650 * 10(-2), 0.168 * 10(-1), 0.863 * 10(-2)$,
$0.147 * 10(-1), 0.492 * 10(-2), 0.359 * 10(-2), 0.803 * 10(-2), 0.894 * 10(-2), 0.575 * 10(-2)$,
$0.981 * 10(-3), 0.264 * 10(-2), 0.160 * 10(-2), 0.394 * 10(-2), 0.297 * 10(-2), 0.167 * 10(-2)$,
$0.460 * 10(-2), 0.235 * 10(-2), 0.380 * 10(-2), 0.531 * 10(-2), 0.381 * 10(-2), 0.494 * 10(-2)$,
$0.501 * 10(-2), 0.317 * 10(-2), 0.336 * 10(-2), 0.170 * 10(-2), 0.209 * 10(-1), 0.955 * 10(-2)$,
$0.468 * 10(-2), 0.202 * 10(-2), 1.00), 0.487 * 10(-2), 0.746 * 10(-2])$,
$[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0$,
$0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0]$,
$[0.320 * 10(-3), 0.169 * 10(-3), 0.269 * 10(-3), 0.305 * 10(-3), 0.623 * 10(-3), 0.147 * 10(-3)$,
$0.399 * 10(-3), 0.874 * 10(-3), 0.994 * 10(-3), 0.195 * 10(-2), 0.114 * 10(-2), 0.847 * 10(-3)$,
$0.535 * 10(-3), 0.815 * 10(-3), 0.934 * 10(-3), 0.758 * 10(-3), 0.132 * 10(-2), 0.755 * 10(-3)$,
$0.972 * 10(-3), 0.114 * 10(-2), 0.935^{*} 10(-3), 0.125 * 10(-2), 0.131 * 10(-2), 0.249 * 10(-3)$,
$0.831 * 10(-3), 0.113 * 10(-2), 0.534 * 10(-3), 0.732 * 10(-3), 0.237 * 10(-3), 0.463 * 10(-3)$,
$0.312 * 10(-3), 0.495 * 10(-3), 0.147 * 10(-3), 0.320 * 10(-3), 0.360 * 10(-3), 0.111 * 10(-2)$,
$0.118 * 10(-2), 0.476 * 10(-3), 0.174 * 10(-2), 0.760 * 10(-3), 0.736 * 10(-3), 0.441 * 10(-3)$,
$0.165 * 10(-3), 0.240 * 10(-3), 0.576 * 10(-3), 0.169 * 10(-2), 0.118 * 10(-2), 0.442 * 10(-3)$,
$0.885^{*} 10(-3), 0.555^{*} 10(-3), 0.111 * 10(-2), 0.330 * 10(-2), 0.384^{*} 10(-3), 0.162 * 10(-2)$,
$0.654 * 10(-3), 0.903 * 10(-2), 0.748 * 10(-3), 0.729 * 10(-3), 0.409 * 10(-2), 0.123 * 10(-2)$,
$0.103 * 10(-2), 0.103 * 10(-2), 0.394 * 10(-2), 0.272 * 10(-2), 1.03]]$

## Conclusion

Most of entries of matrix $(I-A)^{-1}$ are positive and the rest of entries are close to 0 ), so the economic is productive.

