

Fractals and Linear Algebra

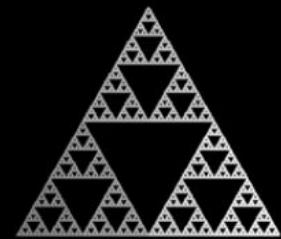
Bryan Clark; Noah Jackson; Lindsay Finlayson; Garrett
Bullough

Thin
wire

Flat
sheet

Solid
cube

Sierpinski
mesh



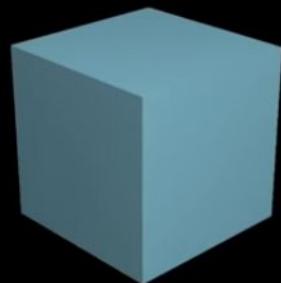
Line



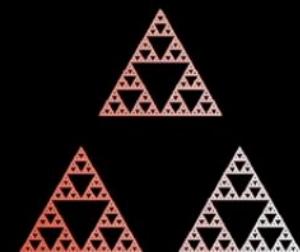
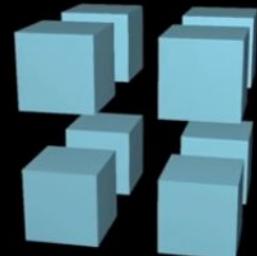
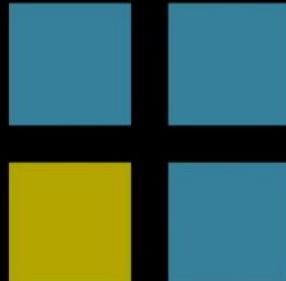
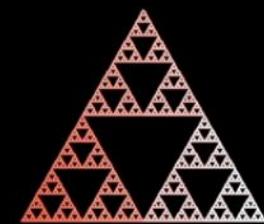
Square



Cube



Sierpinski



$1/2$ length

$1/4$ area

$1/8$ volume

Line

Scaling factor: $\frac{1}{2}$



Mass scaling factor: $\frac{1}{2}$



Square

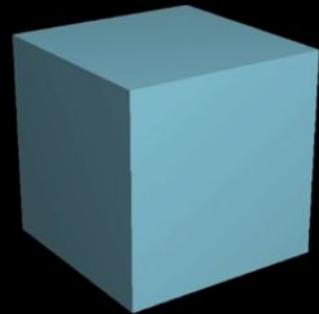
$$1 \left\{ \begin{array}{c} \text{[Blue square]} \\ \text{[Blue square]} \end{array} \right.$$

Scaling factor: $\frac{1}{2}$

Mass scaling factor: $\frac{1}{4} = \left(\frac{1}{2}\right)^2$

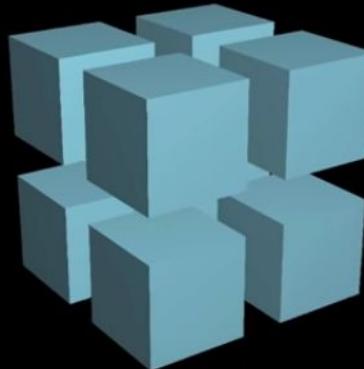
$$\frac{1}{2} \left\{ \begin{array}{c} \text{[Blue square]} \\ \text{[Blue square]} \end{array} \right.$$

Cube



Scaling factor: $\frac{1}{2}$

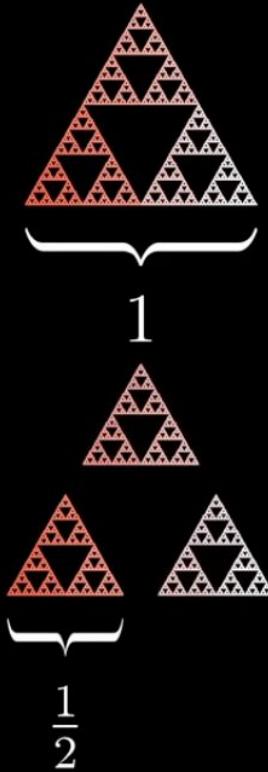
Mass scaling factor: $\frac{1}{8} = \left(\frac{1}{2}\right)^3$



Sierpinski

Scaling factor: $\frac{1}{2}$

Mass scaling factor: $\frac{1}{3}$



2-dimensional

Length: L



Mass: M

Length: sL

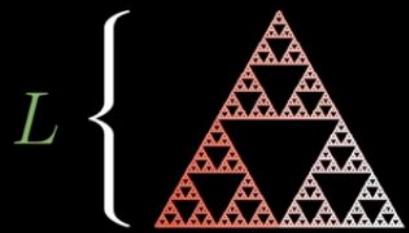


Mass: $s^2 M$

D-dimensional

$$\left(\frac{1}{2}\right)^D = \left(\frac{1}{3}\right)$$

Length: L



Mass: M

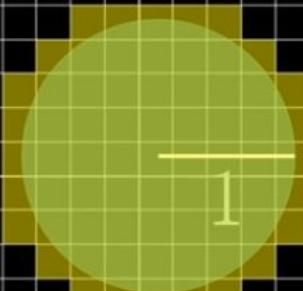
$$2^D = 3$$

$$\log_2(3) \approx 1.585$$

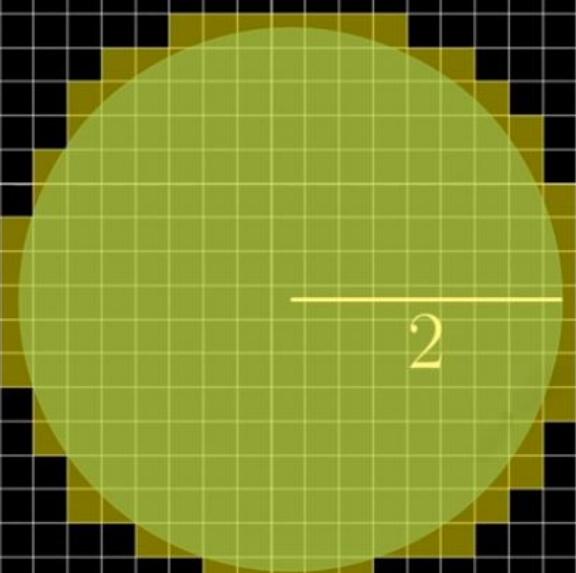
Length: $\left(\frac{1}{2}\right)L$ $\left(\frac{1}{2}\right)L \left\{$ $\right\}$

Mass: $\left(\frac{1}{2}\right)^D M$

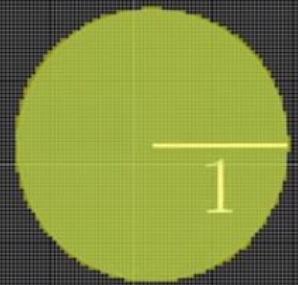
Boxes touched: 69



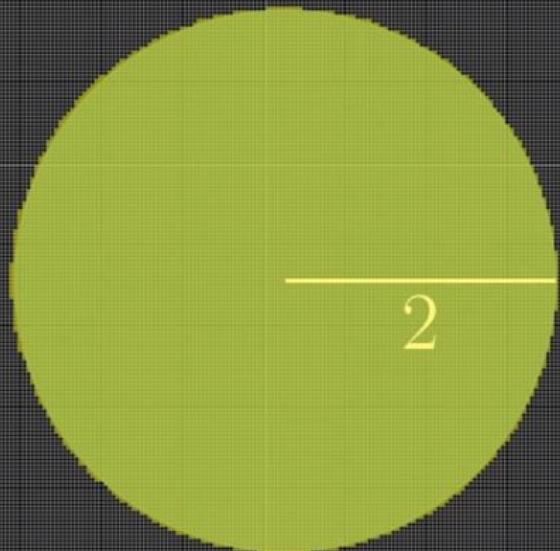
Boxes touched: $\frac{234}{69} = 3.39 \approx 2^2$

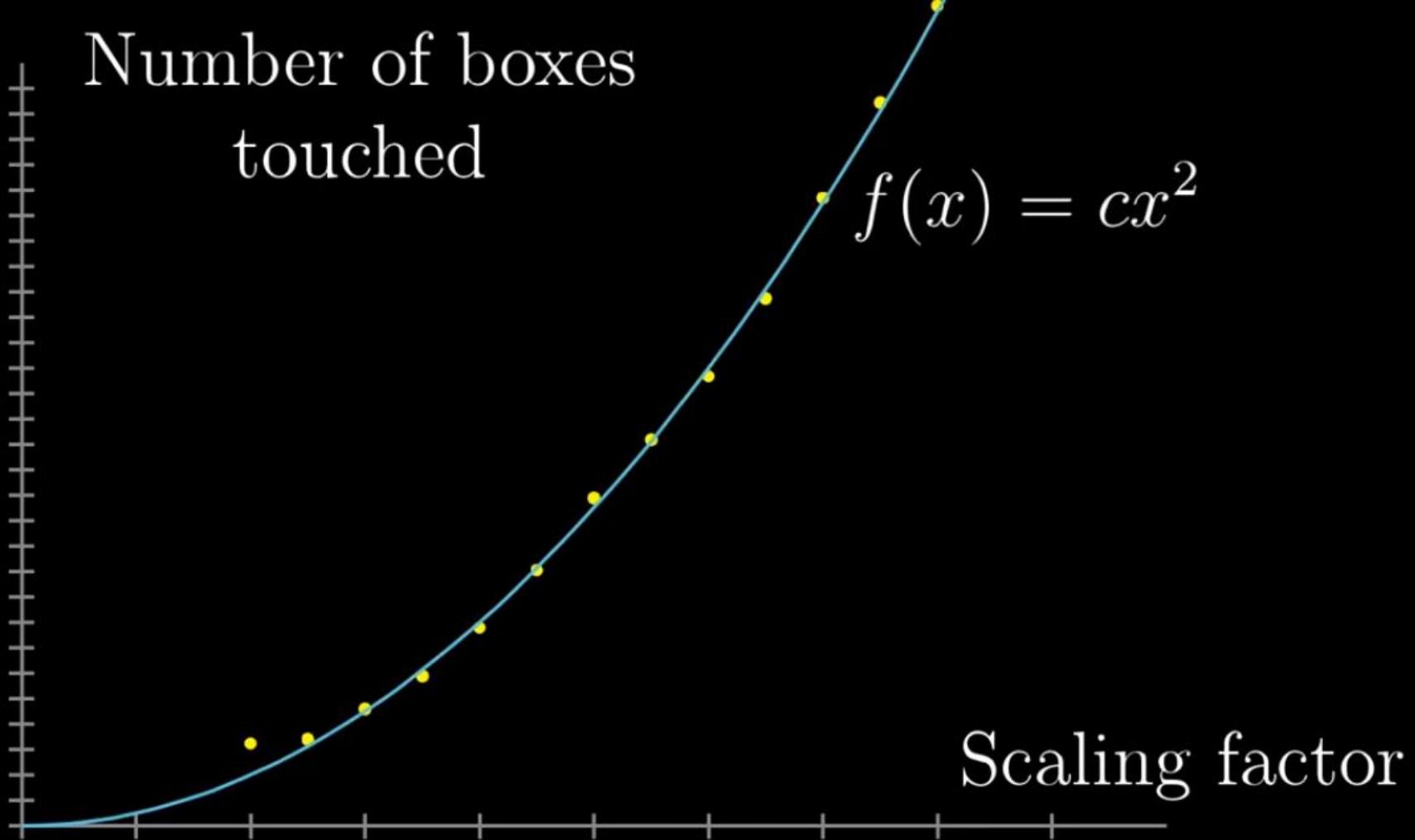


Boxes touched: 3624

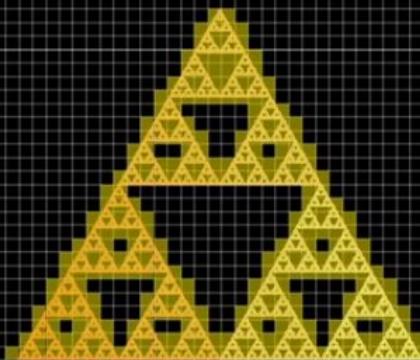


Boxes touched: $\frac{14235}{3624} = 3.93 \approx 2^2$

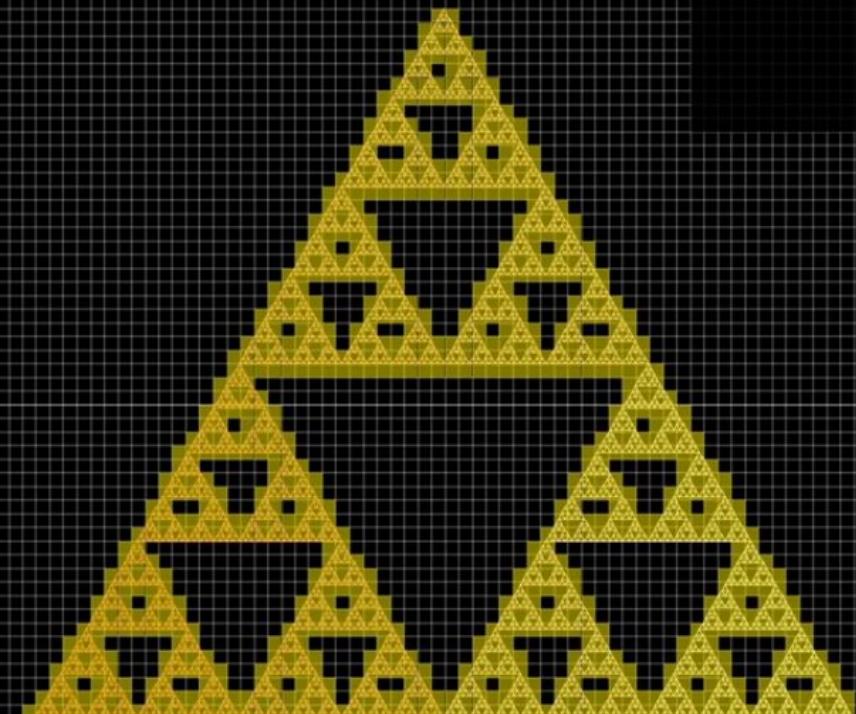




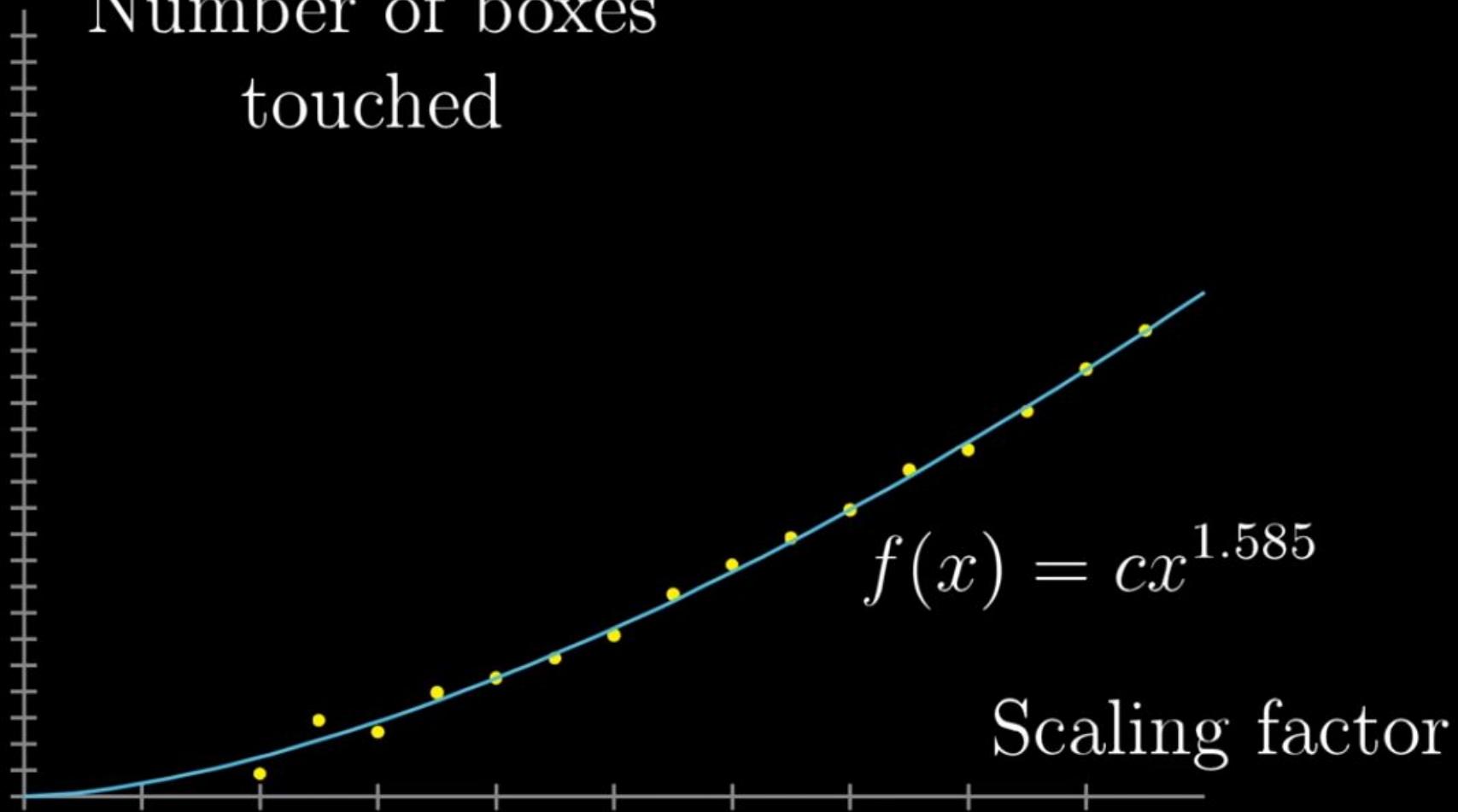
Boxes touched: 302



Boxes touched: $\frac{928}{302} \approx 3 = 2^{1.585\dots}$



Number of boxes
touched









Number of boxes \approx
 $c(\text{scaling factor})^{1.21}$



$$N \approx c s^{1.21}$$
$$\log(N) = \log(c s^{1.21})$$



$$N \approx c s^{1.21}$$

$$\log(N) = \log(c) + 1.21 \log(s)$$

$\log(N)$

$$N \approx c s^{1.21}$$

$$\log(N) = \log(c) + 1.21 \log(s)$$

Slope = 1.21

$\log(s)$



Britain coast:
1.21-dimensional



Norway coast:
1.52-dimensional

Dimension ≈ 2.05



Dimension ≈ 2.3



Is it a
fractal?

Yes

Probably from
nature

No

Probably
man-made

References: <https://www.youtube.com/watch?v=gB9n2gHsHN4>

