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> with(LinearAlgebra):
> A := <0,1|1,1>;Eigenvectors(A);
```

$$A := \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{c} \frac{\sqrt{5}}{2} + \frac{1}{2} \\ \frac{1}{2} - \frac{\sqrt{5}}{2} \end{array} \right], \left[\begin{array}{cc} \frac{1}{\frac{\sqrt{5}}{2} + \frac{1}{2}} & \frac{1}{\frac{1}{2} - \frac{\sqrt{5}}{2}} \\ 1 & 1 \end{array} \right]$$

(1)

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> v1:=<1,c>;v2:=<1,d>;D1:=<c,0|0,d>^+;P:=<v1|v2>;
```

$$D1 := \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$$

$$P := \begin{bmatrix} 1 & 1 \\ c & d \end{bmatrix}$$

(2)

```
> simplify(subs(c=(1+sqrt(5))/2,d=(1-sqrt(5))/2,A.P-P.D1));
```

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(3)

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> D2:=<c^k,0|0,d^k>^+;
```

$$D2 := \begin{bmatrix} c^k & 0 \\ 0 & d^k \end{bmatrix}$$

(4)

```
> Ak:=simplify(P .D2 . (1/P));
```

$$Ak := \begin{bmatrix} \frac{-c^k d + d^k c}{c - d} & \frac{c^k - d^k}{c - d} \\ \frac{-c^{1+k} d + d^{1+k} c}{c - d} & \frac{c^{1+k} - d^{1+k}}{c - d} \end{bmatrix}$$

(5)

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> # Because cd=-1, this reduces to 1/(c-d) times the matrix
> tmp:=simplify(<-d*c^k+c*d^k,c^k-d^k|c^k-d^k,c*c^k-d*d^k>^+);
```

$$tmp := \begin{bmatrix} -c^k d + d^k c & c^k - d^k \\ c^k - d^k & c^{1+k} - d^{1+k} \end{bmatrix}$$

(6)

```
> simplify(subs(c=(1+sqrt(5))/2,d=(1-sqrt(5))/2,c*d));
```

-1

(7)

```
> simplify(subs(c=(1+sqrt(5))/2,d=(1-sqrt(5))/2,c-d));
```

$\frac{\sqrt{5}}{2}$

(8)

```
> simplify(subs(c=(1+sqrt(5))/2,d=(1-sqrt(5))/2,c+d));
```

1

(9)

```
> # Problem 7 details
# uk := Ak . u0; uk[1] is component 1, which is yk. Vector u0:=
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<1,1>;
# Use cd=-1 and c+d=1 to simplify yk = uk[1] = -c^k * d + d^k * c
+ c^k - d^k
# into two terms. Collect as follows: c^k*(1-d)+d^k*(c-1).
# Then replace 1-d=c and c-1=-d. The result: c^{k+1}-d^{k+1}
> # Divide by c-d.
> ans7:=(c^(k+1)-d^(k+1))/(d-c);

```

$$ans7 := \frac{c^{k+1} - d^{k+1}}{d - c} \quad (10)$$

```

> # Replace c by lambda1 and d by lambda2
> subs(c=(1+sqrt(5))/2,d=(1-sqrt(5))/2,ans7);

```

$$- \frac{\left(\left(\frac{1}{2} + \frac{\sqrt{5}}{2} \right)^{k+1} - \left(\frac{1}{2} - \frac{\sqrt{5}}{2} \right)^{k+1} \right) \sqrt{5}}{5} \quad (11)$$