MATH 2270-4 Sample Exam 1 S2019

No books or notes. No electronic devices, please.
These problems have credits 10 to 25, which is an estimate of the time required to write the solution.
This sample will be edited during the semester to reflect course content. Expect to see problems that appeared on the 2018 final exam.
The actual exam will have the same number of problems, identical problem types, on exactly the same topics. Covered on the exam are chapters 1, 2 and parts of 3 from the 2270 textbook [**Lay** et al]. This sample edited Feb 12 to remove determinant topics.

Problem 1. (10 points)
(a) Give a counter example or explain why it is true. If A and B are $n \times n$ invertible, and C^T denotes the transpose of a matrix C, then $(AB^{-1})^T = (B^T)^{-1}A^T$.
(b) Give a counter example or explain why it is true. If square matrices A and B satisfy $AB = I$, then $BA = I$ and $A^TB^T = I$.

Problem 2. (10 points) Let A be a 3×4 matrix. Find the elimination matrix E which under left multiplication against A performs both (1) and (2) with one matrix multiply.

(1) Replace Row 2 of A with Row 2 minus Row 3.

(2) Replace Row 3 of A by Row 3 minus 4 times Row 1.

Problem 3. (30 points) Let a, b and c denote constants and consider the system of equations

$$\begin{pmatrix} 1 & b & c \\ 1 & c & -a \\ 2 & b+c & a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -a \\ a \\ a \end{pmatrix}$$

Use techniques learned in this course to briefly explain the following facts. Only write what is needed to justify a statement.

(a). The system has a unique solution for $(c-b)(2a-c) \neq 0$.

(b). The system has no solution if $c = 2a$ and $a \neq 0$ (don't explain the other possibilities).

(c). The system has infinitely many solutions if $a = b = c = 0$ (don't explain the other possibilities).

Definition. *Vectors* $\vec{v}_1, \ldots, \vec{v}_k$ *are called* **independent** *provided solving the equation* $c_1\vec{v}_1 + \cdots + c_k\vec{v}_k = \vec{0}$ *for constants* $c_1, \ldots, c_k$ *has the unique solution* $c_1 = \cdots = c_k = 0$. *Otherwise the vectors are called* **dependent**.

Problem 4. (20 points) Classify the following sets of vectors as Independent or Dependent, using the Pivot Theorem or the definition of independence (above).

$$\text{Set 1:} \quad \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$\text{Set 2:} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

Problem 5. (20 points) Find the vector general solution $\vec{x}$ to the equation $A\vec{x} = \vec{b}$ for

$$A = \begin{pmatrix} 1 & 0 & 0 & 4 \\ 3 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$$

Problem 6. (20 points) Determinant problem, chapter 3. Parts reduced on Exam 1.
(a) [10%] True or False? The value of a determinant is the product of the diagonal elements.
(b) [10%] True or False? The determinant of the negative of the $n \times n$ identity matrix is -1.
(c) [30%] Supply proof details or else display a counterexample: If A, B are 3×3 matrices and both have an inverse, then $|(A+B)^{-1}| = |A^{-1}| + |B^{-1}|$.
(d) [50%] Determine all values of x for which $(2I + C)^{-1}$ fails to exist, where I is the 3×3 identity and $C = \begin{pmatrix} 2 & x & -1 \\ 3x & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix}$.

End Exam 1.