

Sample Quiz3 Problem 1. A graphic called a **phase diagram** displays the behavior of all solutions of $u' = F(u)$. A **phase line diagram** is an abbreviation for a direction field on the vertical axis (u -axis). It consists of equilibrium points and signs of $F(u)$ between equilibria. A phase diagram can be created solely from a phase line diagram, using just three drawing rules:

1. Solutions don't cross.
2. Equilibrium solutions are horizontal lines $u = c$. All other solutions are increasing or decreasing.
3. A solution curve can be moved rigidly left or right to create another solution curve.

Use these tools on the equation $u' = u(u^2 - 4)$ to make a phase line diagram, and then make a phase diagram with at least 8 threaded solutions. Label the equilibria as stable, unstable, funnel, spout, node.

References. Edwards-Penney section 2.2. Course document on Stability,

Sample Quiz3 Problem 2. An autonomous differential equation $\frac{dy}{dx} = F(x)$ with initial condition $y(0) = y_0$ has a formal solution

$$y(x) = y_0 + \int_0^x F(u) du.$$

The integral may not be solvable by calculus methods. In this case, the integral is evaluated numerically to compute $y(x)$ or to plot a graphic. There are three basic numerical methods that apply, the rectangular rule (RECT), the trapezoidal rule (TRAP) and Simpson's rule (SIMP).

Apply the three methods for $F(x) = \sin(x^2)$ and $y_0 = 0$ using step size $h = 0.2$ from $x = 0$ to $x = 1$. Then fill in the blanks in the following table. Use technology if it saves time. Lastly, compare the four data sets in a plot, using technology.

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|--------------------|-----|----------------------|----------------|----------------------|----------------------|----------------------|
| x - values | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
| y - to 10 digits | 0.0 | 0.0026663619 | 0.02129435557 | 0.07133622797 | 0.1657380596 | 0.3102683017 |
| y - RECT values | 0.0 | 0.0 | 0.007997866838 | <input type="text"/> | <input type="text"/> | 0.2297554431 |
| y - TRAP values | 0.0 | <input type="text"/> | 0.02392968750 | 0.07508893150 | <input type="text"/> | 0.3139025416 |
| y - SIMP values | 0.0 | 0.002666288917 | 0.02129368017 | <input type="text"/> | 0.1657330636 | <input type="text"/> |

References. Edwards-Penney Sections 2.4, 2.5, 2.6, because methods Euler, Modified Euler and RK4 reduce to RECT, TRAP, SIMP methods when $f(x, y)$ is independent of y , i.e., an equation $y' = F(x)$. Course document on numerical solution of $y' = F(x)$ RECT, TRAP, SIMP methods. Wolfram Alpha at <http://www.wolframalpha.com/> can do the RECT rule and graphics with input string

`integrate sin(x^2) using left endpoint method with interval width 0.2 from x=0 to x=1`