```
libname := "/u/ma/gustafson/bin/laylinalg.mla",
    "/usr/local/sys/maple/maple2016/lib"
```
(1)

```
> # Solve for the exact answers
  # System x'=3x-2y, y'=5x-4y, x(0)=3, y(0)=6
  de1:=diff(x(t),t)=3*x(t)-2*y(t);de2:=diff(y(t),t)=5*x(t)-4*y(t);
  ic:=x(0)=3,y(0)=6;
  dsolve([de1,de2,ic],[x(t),y(t)]);
```

$$de1 := \frac{d}{dt} x(t) = 3\,x(t) - 2\,y(t)$$

$$de2 := \frac{d}{dt} y(t) = 5\,x(t) - 4\,y(t)$$

$$ic := x(0) = 3,\ y(0) = 6$$

$$\{x(t) = 2\,e^{-2\,t} + e^{t},\ y(t) = 5\,e^{-2\,t} + e^{t}\}$$
(2)

```
> Ans:=t->evalf(<2*exp(-2*t)+exp(t), 5*exp(-2*t)+exp(t)>); # Exact
  answers
```

$$Ans := t \to evalf(\langle 2\,e^{-2\,t} + e^{t},\ 5\,e^{-2\,t} + e^{t}\rangle)$$
(3)

```
> # Approximate methods are Euler, Heun, RK4.
  # Notation for t-values: t0=initial point, t1=t0+h, t2=t1+h, ...
  # Notation for u-values: 1u0=u(t0)=initial values, u1 approx u
  (t1), u2 approx u(t2), ....

> # Solve for approximate answers using Euler's Method
  F:=(t,u)-><3*u[1]-2*u[2],5*u[1]-4*u[2]>;
```

$$F := (t, u) \to \langle 3\,u_1 - 2\,u_2,\ 5\,u_1 - 4\,u_2\rangle$$
(4)

```
> h:=0.1; t0:=0; u0:=<3,6>;
```

$$h := 0.1$$

$$t0 := 0$$

$$u0 := \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$
(5)

```
> u1:=u0+h*F(t0,u0);t1:=t0+h;Ans(t1);
```

$$u1 := \begin{bmatrix} 2.700000000000000 \\ 5.100000000000000 \end{bmatrix}$$

$$t1 := 0.1$$

$$\begin{bmatrix} 2.742632424 \\ 5.198824684 \end{bmatrix}$$
(6)

```
> u2:=u1+h*F(t1,u1);t2:=t1+h;Ans(t2);
```

$$u2 := \begin{bmatrix} 2.490000000000000 \\ 4.410000000000000 \end{bmatrix}$$

$$t2 := 0.2$$

$$\begin{bmatrix} 2.562042850 \\ 4.573002988 \end{bmatrix} \quad (7)$$

```
> # Solve for approximate answers using Heun's Method
  h:=0.1; t0:=0; u0:=<3,6>;
```

$$h := 0.1$$

$$t0 := 0$$

$$u0 := \begin{bmatrix} 3 \\ 6 \end{bmatrix} \quad (8)$$

```
> w:=u0+h*F(t0,u0);t1:=t0+h;u1:=u0+h*(F(t0,u0)+F(t1,w))/2;Ans(t1);
```

$$w := \begin{bmatrix} 2.70000000000000 \\ 5.10000000000000 \end{bmatrix}$$

$$t1 := 0.1$$

$$u1 := \begin{bmatrix} 2.74500000000000 \\ 5.20500000000000 \end{bmatrix}$$

$$\begin{bmatrix} 2.742632424 \\ 5.198824684 \end{bmatrix} \quad (9)$$

```
> w:=u1+h*F(t1,u1);t2:=t1+h;u2:=u1+h*(F(t1,u1)+F(t2,w))/2;Ans(t2);
```

$$w := \begin{bmatrix} 2.52750000000000 \\ 4.49550000000000 \end{bmatrix}$$

$$t2 := 0.2$$

$$u2 := \begin{bmatrix} 2.56582500000000 \\ 4.58302500000000 \end{bmatrix}$$

$$\begin{bmatrix} 2.562042850 \\ 4.573002988 \end{bmatrix} \quad (10)$$

```
> # Solve for approximate answers using the RK4 Method
  h:=0.1; t0:=0; u0:=<3,6>;
```

$$h := 0.1$$

$$t0 := 0$$

$$u0 := \begin{bmatrix} 3 \\ 6 \end{bmatrix} \quad (11)$$

```
> k1:=h*F(t0,u0);t1:=t0+h;
  k2:=h*F(t0+h/2,u0+k1/2);
```

```
k3:=h*F(t0+h/2,u0+k2/2);
k4:=h*F(t1,u0+k3);
u1:=u0+(k1+2*k2+2*k3+k4)/6;Ans(t1);
```

$$k1 := \begin{bmatrix} -0.300000000000000 \\ -0.900000000000000 \end{bmatrix}$$

$$t1 := 0.1$$

$$k2 := \begin{bmatrix} -0.255000000000000 \\ -0.795000000000000 \end{bmatrix}$$

$$k3 := \begin{bmatrix} -0.258750000000000 \\ -0.804750000000000 \end{bmatrix}$$

$$k4 := \begin{bmatrix} -0.216675000000000 \\ -0.707475000000000 \end{bmatrix}$$

$$u1 := \begin{bmatrix} 2.74263750000000 \\ 5.19883750000000 \end{bmatrix}$$

$$\begin{bmatrix} 2.742632424 \\ 5.198824684 \end{bmatrix}$$

(12)

```
> k1:=h*F(t1,u1);t2:=t1+h;
  k2:=h*F(t1+h/2,u1+k1/2);
  k3:=h*F(t1+h/2,u1+k2/2);
  k4:=h*F(t1,u1+k3);
  u2:=u1+(k1+2*k2+2*k3+k4)/6;Ans(t2);
```

$$k1 := \begin{bmatrix} -0.216976250000000 \\ -0.708216250000000 \end{bmatrix}$$

$$t2 := 0.2$$

$$k2 := \begin{bmatrix} -0.178701062500000 \\ -0.620817062500000 \end{bmatrix}$$

$$k3 := \begin{bmatrix} -0.181699703125000 \\ -0.628728103125000 \end{bmatrix}$$

$$k4 := \begin{bmatrix} -0.145740540312500 \\ -0.547574860312500 \end{bmatrix}$$

$$u2 := \begin{bmatrix} 2.56205111307292 \\ 4.57302392640625 \end{bmatrix}$$

$$\begin{bmatrix} 2.562042850 \\ 4.573002988 \end{bmatrix} \tag{13}$$