Differential Equations 2280

Midterm Exam 2 Review: Problems Only
Exam Date: 30 March 2018 at 12:50pm

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

1. **(Chapter 3)**
   (a) [60%] Find by any applicable method the steady-state periodic solution.
   (b) [40%] Variation of parameters: electrical/mechanical system.

   Expected in (b): (1) Characteristic equation and Euler atoms. (2) Wronskian, (3) Explicit formula for y_h. (4) Integral formula for y_p, simplified as much as possible, integrals unevaluated.

2. **(Laplace Theory)**
   (a) [50%] Assume $f(t)$ is of exponential order. Find $f(t)$ in the given relation.

   (b) [50%] Solve by Laplace's method: second order forced spring-mass system with initial conditions.

3. **(Laplace Theory)**
   (a) [40%] Solve a linear third order homogeneous differential equation by Laplace's Method.

   Expected in (a): (1) Laplace's method steps to find $\mathcal{L}(x(t))$, (2) Backward table steps to find $x(t)$, (3) An answer check.

   (b) [60%] Solve a 2×2 system of non-homogeneous linear differential equations by Laplace's Method.

   Expected in (b): (1) Laplace's method steps to find a system of linear algebraic equations for $\mathcal{L}(x(t))$ and $\mathcal{L}(y(t))$. (2) Cramer's Rule steps to find $\mathcal{L}(x(t))$ and $\mathcal{L}(y(t))$. (3) Backward Laplace table steps to find $x(t), y(t)$. (4) Check the answer for sanity, e.g., compute the Euler atoms.

4. **(Systems of Differential Equations)**
   The Eigenanalysis Method (section 5.2) says that, for a 3×3 system $\frac{d}{dt}\vec{u}(t) = A\vec{u}(t)$, the general solution is $\vec{u}(t) = c_1\vec{v}_1 e^{\lambda_1 t} + c_2\vec{v}_2 e^{\lambda_2 t} + c_3\vec{v}_3 e^{\lambda_3 t}$. In the solution formula, $(\lambda_1, \vec{v}_1)$, $(\lambda_2, \vec{v}_2)$, $(\lambda_3, \vec{v}_3)$ are eigenpairs of A. Assume given the two 3×3 matrices

$$A = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}, \qquad B = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}.$$

   (a) [50%] Matrix A has only two eigenpairs. Display eigenanalysis details for A. Then explain method failures in methods (1) to (4).

   (b) [50%] Find by any method the general solution of $\frac{d}{dt}\vec{u}(t) = B\vec{u}(t)$.

5. **(Systems of Differential Equations)**
   A system $\frac{d}{dt}\vec{u} = A\vec{u}$ with A an $n \times n$ real matrix can be solved by many methods. Consider below **only these possibilities**:

   (1) Cayley-Hamilton-Ziebur method, from section 4.2. (2) Eigenanalysis method from section 5.2. (3) Laplace's method, from chapter 7. (4) First order linear integrating factor method from section 1.5.

(a) [20%] Given an example system, which of methods (1), (3), (4) might be possible to use? Don't do any details or illustrations: kindly write a sentence and explain your logic.

(b) [40%] Solve for the general solution by the Cayley-Hamilton-Ziebur shortcut (chapters 4 and 5):

$$\frac{d}{dt}\vec{u} = B\vec{u}, \qquad B = \begin{bmatrix} * & * \\ * & * \end{bmatrix} \qquad \textbf{(eigenvalues} = *, *).$$

(c) [40%] Choose any method applicable and solve for the general solution of system

$$\begin{cases} x'(t) &= *x(t) + *y(t) + *z(t), \\ y'(t) &= *x(t) + *y(t) + *z(t), \\ z'(t) &= *x(t) + *y(t) + *z(t). \end{cases}$$