Differential Equations 2280

Midterm Exam 2 Problems Only
Exam Date: 31 March 2017 at 12:50pm

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

1. **(Chapter 3)**
   (a) [70%] Find by any applicable method the steady-state periodic solution for the current equation $I'' + 2I' + 5I = 10\cos(t) - 100\sin(t)$.

   (b) [30%] Linear algebra can find the solution of the current equation $I''+2I'+5I = 10\cos(t)-100\sin(t)$ having initial conditions $I(0) = 1$, $I'(0) = 0$. Write the linear algebraic equations for c_1, c_2, but to save time don't solve for c_1, c_2.

2. **(Laplace Theory)**
   (a) [40%] Assume $f(t)$ is of exponential order. Find $f(t)$ in the relation

$$\frac{d^2}{ds^2}\mathcal{L}(f(t))\bigg|_{s\to(s-3)} = \frac{1}{s^2} + \mathcal{L}(t^2 f(t) - t).$$

   (b) [60%] Solve by Laplace's method $x'' + 2x' + x = e^{-t}$, $x(0) = x'(0) = 0$.

3. **(Laplace Theory)**
   (a) [30%] Solve $\mathcal{L}(f(t)) = \dfrac{10/s}{(s^2+1)(s^2+5)}$ for $f(t)$.

   (b) [30%] Solve $x''' + x' = 0$, $x(0) = 1$, $x'(0) = 1$, $x''(0) = 0$ by Laplace's Method.

   (c) [40%] Solve the system $x' = 4x + y + 30$, $y' = x + 4y + 60$, $x(0) = 0$, $y(0) = 0$ by Laplace's Method.

4. **(Systems of Differential Equations)**
   The Eigenanalysis Method (section 5.2) says that, for a 3×3 system $\frac{d}{dt}\vec{u} = A\vec{u}$, the general solution is $\vec{u}(t) = c_1\vec{v}_1 e^{\lambda_1 t} + c_2\vec{v}_2 e^{\lambda_2 t} + c_3\vec{v}_3 e^{\lambda_3 t}$. In the solution formula, $(\lambda_1, \vec{v}_1)$, $(\lambda_2, \vec{v}_2)$, $(\lambda_3, \vec{v}_3)$ are eigenpairs of A. Assume given the 3×3 matrix

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 0 & 4 & 1 \\ 0 & 0 & 5 \end{bmatrix}.$$

   (a) [50%] Matrix A has only two eigenpairs. Display eigenanalysis details for A.
   (b) [25%] It is impossible to apply the Eigenanalysis Method (stated above). Explain why.
   (c) [25%] Display the solution of $\frac{d}{dt}\vec{u} = A\vec{u}$ in case A is 4×4 and has eigenvalues $2, -1, 3, 5$ with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}.$$

5. **(Systems of Differential Equations)**
   Systems $\frac{d}{dt}\vec{u} = A\vec{u}$ with A an $n \times n$ real matrix can be solved by the following methods:

   (1) Cayley-Hamilton-Ziebur method, from section 4.2. (2) Eigenanalysis method from 5.2. (3) Laplace's method, from chapter 7. (4) Exponential matrix, from 5.6

(a) [50%] The eigenvalues are 4, 6 for the matrix $A = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}$. Display the general solution of $\frac{d}{dt}\vec{u} = A\vec{u}$ according to the Cayley-Hamilton-Ziebur shortcut (textbook chapters 4,5).

(b) [10%] The 3×3 system $\frac{d}{dt}\vec{u} = A\vec{u}$ is supplied with matrix A having only two eigenpairs. It can be solved using the exponential matrix. What other methods are possible to use? Don't do any details, write a sentence.

(c) [10%] The 3×3 system $\frac{d}{dt}\vec{u} = A\vec{u}$ is supplied with matrix A having three eigenpairs, but only one real eigenvalue. It can be solved using the exponential matrix. What other methods are possible to use? Don't do any details, write a sentence.

(d) [30%] The 3×3 system $\frac{d}{dt}\vec{u} = A\vec{u}$ is given by $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. Choose a method other than the exponential matrix and explain how you would solve for $\vec{u}$. It is not necessary to find the answer, but it is necessary to outline the method, not omitting any details.