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Fractals: A Visual Display of Mathematics
Linear Algebra - Math 2270

Introduction:

Fractals are infinite patterns that look similar at all levels of magnification and exist between the normal dimensions. With the advent of the computer, we can generate these complex structures to model natural structures around us such as blood vessels, heartbeat rhythms, trees, forests, and mountains, to name a few. I will begin by explaining how different linear transformations have been used to create fractals. Then I will explain how I have created fractals using linear transformations and include the computer-generated results.

A Brief History:

Fractals seem to be a relatively new concept in mathematics, but that may be because the term was coined only 43 years ago. It is in the century before Benoit Mandelbrot coined the term that the study of concepts now considered fractals really started to gain traction. The invention of the computer provided the computing power needed to generate fractals visually and further their study and interest.

Expand on the ideas by century:

17th century ideas

- Leibniz

19th century ideas

- Karl Weierstrass
- George Cantor
- Felix Klein
- Henri Poincare

20th century ideas

- Helge von Koch
- Waclaw Sierpinski
- Gaston Julia
- Pierre Fatou
- Felix Hausdorff
- Paul Levy
- Benoit Mandelbrot
- Lewis Fry Richardson
- Loren Carpenter

How They Work:

Infinitely complex objects, revealed upon enlarging.

Basics: translations, uniform scaling and non-uniform scaling then translations. Utilize translation vectors.

Concepts used in fractals-- Affine Transformation (operate on individual points in the set), Rotation Matrix, Similitude Transformation

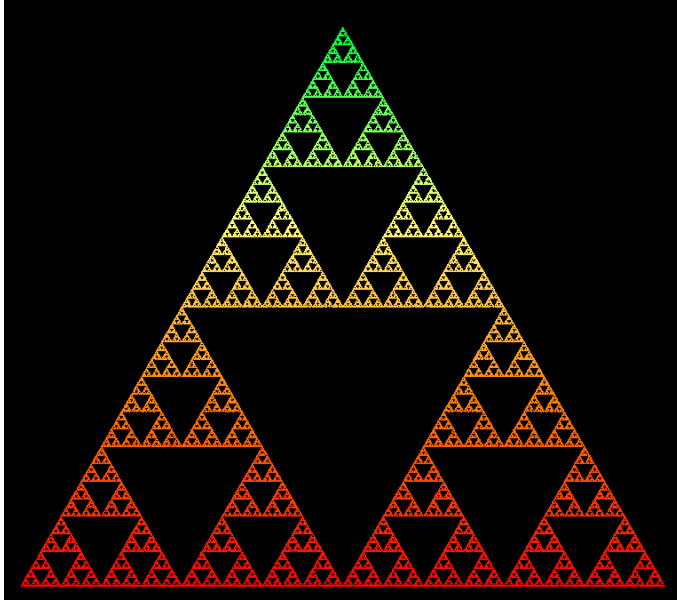
Affine-- translations, scalings, reflections, rotations

Insert Equations here.

Example of how they work with the Koch Curve—curve gets longer with every iteration, and each segment of the iteration is replaced with the pattern from the previous iteration over and over.

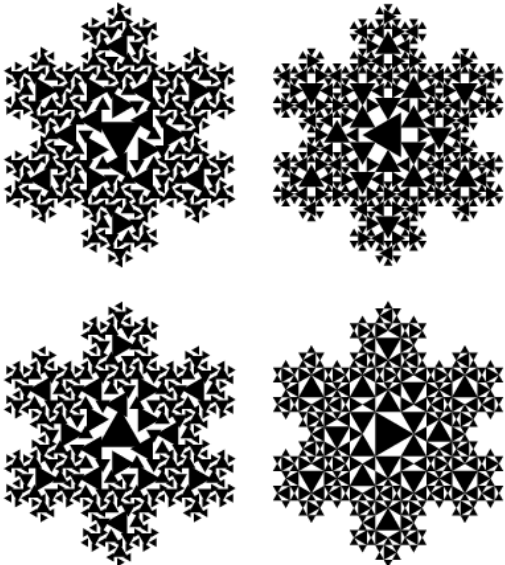
Types of Fractals:

Sierpinski - Recursion of dividing a triangle into smaller divisions of triangles creating an image composed of triangles within each other and layered next to each other.



<http://ecademy.agnesscott.edu/~lriddle/ifs/siertri/siertri.htm>

Koch - Recursion curve with triangle like formations branching off from one another. The inside of the shapes tend to stay hollow with only the outer layer showing.



<http://mathworld.wolfram.com/KochSnowflake.html>

Galaxies - The clusters within the sky from stars, meteorites, and the mysteries of space bring a fractal like formation to the galaxy.



Fractals in Nature:

We may not realize, but many objects in nature have fractal-like structures. A repeated simple branching process is an integral part of the natural world around us. Just as a fractal is formed by a simple pattern repeated in a loop, many objects in nature are formed by a similar repetitive and combining expansion.

Examples:

- Geographic features
- Romanesco
- Lightning, hurricanes
- Trees/ferns
- Kidneys, blood vessels

Bibliography

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