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Productivity of the 2008 Economy

In 2008, the US market crashed and started to be rebuilt again. A consumption matrix will show certain traits of the 2008 economy. The examination of the 2008 consumption matrix will be beneficial to determine patterns and the depth of the issues within the economy.

Sums of the matrix columns will tell if the sectors were profitable. Each column in the consumption matrix represents a particular sector of the economy. If the sum of the column is less than 1, then this sector of the economy is profitable and one unit of goods costs less to make and sell than the price it is sold for. On the contrary, if the sum is more than one, then the revenue earned for the unit is less than the expenses incurred to make and sell it. Consequently, that sector of the economy is operating at a loss and is not productive. Examining the sums of the given consumption matrix columns can be rather time consuming, especially in large economies.

Another approach to determine whether the economy was productive is to use the Leontief Input-Output Model / Production Equation: $x = Cx + d$ $d = x - Cx$

$x = (I - C)^{-1} d$ (p.136 in Linear Algebra and Its Applications Textbook)

Where C is the given consumption matrix, d is the demand, x is amount produced, and Cx will give us the number of units that are consumed. Calculating $(I - C)^{-1}$ and analyzing the entries will tell if the economy was productive in 2008.

In order to evaluate productivity using solely the consumption matrix, begin by creating the input-output (use) matrix using the data for 2008 from the Bureau of Economic Analysis. To turn this use matrix into a consumption matrix, we need to divide each column by the total industry output vector. The total industry output vector obtained from the BEA spreadsheet can be found in attached maple code. Then, calculate the consumption matrix by the formula C=Use Matrix [i,j]/Total Industry Output Vector [j], $1 \leq i \leq 64$ and $1 \leq j \leq 64$. To analyze, sum each column from said matrix.

Analysis has led to the conclusion that 64 out of 64 sectors of the economy were profitable. All 64 column sums were less than one. Therefore, based on this examination the economy was profitable, in spite of the downturn experienced in 2008. Please find the consumption matrix and column sums in the Maple Source Code attached.

To evaluate using the Leontief Input-Output Model, or production equation, find the determinant of (I-C), which is not 0, so (I-C) is invertible and $(I-C)^{-1}$ exists. Since the demand d and the consumption matrix are nonnegative, $(I-C)^{-1}$ cannot be negative. Finally, calculate $(I-C)^{-1}$ and inspect every entry of this matrix.

The entries are nonnegative and the only negative entries are really close to zero (in fact Maple shows them as "-0."). Hence, we can assume that that the production vector X is nonnegative, and therefore, the economy in 2008 was productive. Please see the inverse entries of (I-C) in Maple Source Code attached.