

Jiwon Nam

Final draft

Image Compression

Introduction

There are tons of images on the Internet, and people are often uploading and downloading the images. However, what if the image file stored as origin file, the image data will need more bytes to be saved if the image quality is higher. In this case, the total amount of digital storage required or data capacity will be extremely high. For this reason, we need to think about image compression to use data capacity efficiently. If there is 1 Megabyte image file with digital matrix of 1000 by 1000 pixel, and one film need 50,000 images, it is required 50 Gigabyte to store one film. It is extremely huge data. To avoid this problem, we need to use image compression.

Matrix in image

For one image file, it must require coordinates to position each pixel values. If there are $n \times m$ pixels which called P in \mathbb{R}^m , $P = [P_1, P_2, \dots, P_n]$. However, each pixel need RGB values which are red green and blue ($A = [A(\text{red}), A(\text{green}), A(\text{blue})]$) that are able to combine to build almost every color. Therefore, to draw image using digital, we need $n \times m \times 3$ matrix to be stored.

What is the SVD

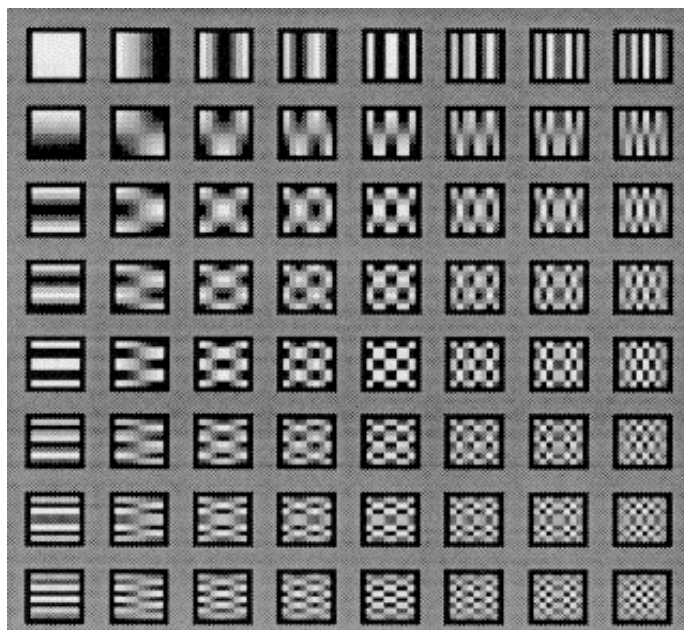
SVD(Singular Value Decomposition) is a rectangular matrix of gene expression data. The SVD theorem states that $A(n \times p) = U(n \times n)S(n \times p)V^T(p \times p)$ where $U^T U = I(n \times n)$, $V^T V = I(p \times p)$. It means that U and V are orthogonal. In this set, U can be made of set of eigenvalues, and V is made of $(A^T) * A$, and S is a square root of eigenvalues from $A * A^T$ or $(A^T) * A$. From this idea, Compression Ratio is around $(1+m+n)/mn$ for mnA . The ratio graph is approaching 1 after specific capacity, and it means that at a special capacity, ratio of compression is really similar. For efficiency of using SVD, a specific capacity will help this process.

What is the DCT

DCT(Discrete Cosine Transformation) is the system that helps separate image in to parts of differing importance. The DCT theorem states that

$F(u) = (2/N)^{1/2} \sum_{i=0}^{N-1} A(i) \cdot \cos[(\pi \cdot u) / 2 \cdot N(2i+1)] f(i)$. The equation of the DCT is complex, but the inverse of $F(u)$ is really simple which is $A(i) = 1/\sqrt{2}$ (if $u = 0$) or 1 (else). In DCT, there is

8 x 8 blocks example which is standard procedure where $N=8$. In this case, the output array of coefficient of DCT contains integer that range from -1024 to 1023. The 8 x 8 blocks has a set of basis functions that can be precomputed and stored. The example of basis set is like this.



Conclusion

SVD(Singular Value Decomposition) is the method of decomposing a matrix into other matrices that helps reducing the dimensionality of data. SVD is very useful system because when several variables that are highly correlated, and when the variables are affected by a small number of underlying, SVD can help to recover some approximate underlying factors. Also, SVD ratio of compression is similar rate after specific capacity so that the speed of computation will be fast with less dimensions can allow usage of algorithms unfit for a bigger dimensions.

DCT(Discrete Cosine Transformation) is the cosine part of the Fourier Transform, and it helps reducing amount of space from input using an array of compressed block that constitute the image. In progress of DCT, at first, whole image of input is divided into small $N \times N$ blocks, and then reducing the storage space using DCT coefficients. Therefore, the large value is decreasing to small, and it also decreasing size of space. The block size (N) affects to quality of compression ratio, so the higher block size get higher compression ratio but it lose more information and quality.