

Applications of Linear Algebra and the Investigative Analysis of Non-Homogenous Stochastics Matrices Modelling Input-Output Flow Models



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The turn of the century brought about a new level of social interconnectedness. Americans moved into overflowing megacities like New York and Chicago. Wages became the norm, not the exception, and subsistence lifestyles were traded in for city living. This new level of monetary interdependence would birth the economy as it is today. As is the case for any fundamental shift in the underlying workings of our lives, the economy warranted study.

Even though the fundamental identities of linear algebra have existed for centuries (in fact the Babylonians possessed a methodology for solving 2×2 linear systems millennia ago), its importance blossomed with its use of calculus, and the identification of flow models that serve a fundamental function to our daily lives; the economy.

20th century economist Wassily Leontif won the Nobel Prize in Economy for his analysis of the monetary input and output of the economy. His analysis and



resulting model of the economy was made possible by linear algebra. By breaking up the economy into major sectors, he was able to set up linear equations in a matrix that represented the income and output of the major sectors of the economy.

The primary goal of our project is to demonstrate and analyze Leontif's economic model using abridged data. As opposed to 500 major economic sectors, the project employs a simplified version of his model using only 5; agriculture, manufacturing, service, government and household. Using real data, compiled from 1947 analytics, a 5×5 system of equations was constructed. The sum of the columns of the matrix represented the output of that sector, and

sum of the rows represented the income of the given sector.

<i>Null</i>	<i>Agriculture</i>	<i>Manufacturing</i>	<i>Services</i>	<i>Government and Other</i>	<i>Household</i>	<i>Total</i>
<i>Agriculture</i>	34.69	5.28	10.45	7.92	26.22	84.5600
<i>Manufacturing</i>	4.92	61.82	25.95	12.45	64.12	169.260
<i>Services</i>	5.62	22.99	42.03	42.53	116.45	229.620
<i>Government and Other</i>	6.27	15.32	16.17	38.99	30.91	107.660
<i>Household</i>	32.97	44.70	114.48	33.09	2.12	227.360
<i>Total</i>	84.4700	150.110	209.080	134.980	239.820	818.460

The matrix was then converted to a stochastic system by dividing each entry by the sum of column of the columns. Because of the way the Maple's linear algebra software operates on floating point numbers, the values were kept in fraction form in order to solve the system.

$\frac{3469}{8447}$	$\frac{528}{15011}$	$\frac{1045}{20908}$	$\frac{396}{6749}$	$\frac{437}{3997}$
$\frac{492}{8447}$	$\frac{6182}{15011}$	$\frac{2595}{20908}$	$\frac{1245}{13498}$	$\frac{458}{1713}$
$\frac{562}{8447}$	$\frac{2299}{15011}$	$\frac{4203}{20908}$	$\frac{4253}{13498}$	$\frac{11645}{23982}$
$\frac{627}{8447}$	$\frac{1532}{15011}$	$\frac{1617}{20908}$	$\frac{3899}{13498}$	$\frac{3091}{23982}$
$\frac{3297}{8447}$	$\frac{4470}{15011}$	$\frac{2862}{5227}$	$\frac{3309}{13498}$	$\frac{106}{11991}$

Following this, in order to appropriately represent a flow model, one was subtracted from the main diagonal because these entries represent closed-system cost and gains.

$-\frac{4978}{8447}$	$\frac{528}{15011}$	$\frac{1045}{20908}$	$\frac{396}{6749}$	$\frac{437}{3997}$
$\frac{492}{8447}$	$-\frac{8829}{15011}$	$\frac{2595}{20908}$	$\frac{1245}{13498}$	$\frac{458}{1713}$
$\frac{562}{8447}$	$\frac{2299}{15011}$	$-\frac{16705}{20908}$	$\frac{4253}{13498}$	$\frac{11645}{23982}$
$\frac{627}{8447}$	$\frac{1532}{15011}$	$\frac{1617}{20908}$	$-\frac{9599}{13498}$	$\frac{3091}{23982}$
$\frac{3297}{8447}$	$\frac{4470}{15011}$	$\frac{2862}{5227}$	$\frac{3309}{13498}$	$-\frac{11885}{11991}$

Finally, an analysis using Maple was performed on the system that determined the rank of the matrix to be four. This indicated an infinite number of solutions and the presence of a one-dimensional nullspace. The nullspace reflects conditions feasible for economic equilibrium.

$$\left[\begin{array}{c} \frac{16875969510185467447}{47615226304432208940} \\ \frac{36100951951338623473}{47615226304432208940} \\ \frac{2267462866834400650}{2380761315221610447} \\ \frac{10252078599554359273}{23807613152216104470} \\ 1 \end{array} \right]$$

Once we added all of the vector values together we were then able to divide the total output of the economy (which we found to be 818.46 billion dollars) by the sum of the vector obtained by the nullspace which gives us our p_{Ho} value. From this we were able to solve for all other values.

$$\begin{bmatrix} p_{Ag} \\ p_{Ma} \\ p_{Se} \\ p_{Go} \\ p_{Ho} \end{bmatrix} = \begin{bmatrix} 82.9839246682410 \\ 177.518611613122 \\ 222.995150151872 \\ 1008.24751753189 \\ 234.137561700000 \end{bmatrix}$$

The purpose of the project was to demonstrate the application of linear algebra in economics. This analysis showed a correlation between fundamental subspaces and the meaning of their representation in fiduciary models. Understanding economics trends and present occurrences is made possible by these means. This is exceedingly important because the modern world is in fact very young, and predictive models are the only means by which we can anticipate dangers in the future.