

Exercise # 1

Write down 3 equations for  $x, y, z$  so that  $x\vec{u} + y\vec{v} + z\vec{w} = \vec{b}$

Can you somehow find the constants  $x, y, z$ ?

100%  
Beautiful  
work!  
-Pr G.

$$\vec{u} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} \quad \vec{w} = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2x \\ -x \\ 0 \end{pmatrix} + \begin{pmatrix} -y \\ 2y \\ -y \end{pmatrix} + \begin{pmatrix} 0 \\ -z \\ 2z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 2x - y + 0 = 1 \\ -x + 2y - z = 0 \\ 0 - y + 2z = 0 \end{cases} \xrightarrow{\text{swap (1,2)}} \begin{cases} -x + 2y - z = 0 \\ 2x - y = 1 \\ -y + 2z = 0 \end{cases} \xrightarrow{\text{mult (1, -1)}} \begin{cases} x - 2y + z = 0 \\ 2x - y = 1 \\ -y + 2z = 0 \end{cases} \xrightarrow{\text{mult (1, -2)}} \begin{cases} x - 2y + z = 0 \\ x - y = 1 \\ -y + 2z = 0 \end{cases} \xrightarrow{\text{sub (1,2)}} \begin{cases} x - 2y + z = 0 \\ x - y = 1 \\ -3y + 2z = 1 \end{cases}$$

$$\begin{cases} x - 2y + z = 0 \\ -x - y + 2z = 0 \\ -3y + 2z = 1 \end{cases} \xrightarrow{\text{swap (2,3)}} \begin{cases} x - 2y + z = 0 \\ -3y + 2z = 1 \\ -x - y + 2z = 0 \end{cases} \xrightarrow{\text{mult (3, 1/4)}} \begin{cases} x - 2y + z = 0 \\ -3y + 2z = 1 \\ -x - y + 2z = 0 \end{cases} \xrightarrow{\text{mult (1, -1)}} \begin{cases} x - 2y + z = 0 \\ -3y + 2z = 1 \\ x + y + 2z = 0 \end{cases} \xrightarrow{\text{sub (1,2)}} \begin{cases} x - 2y + z = 0 \\ -3y + 2z = 1 \\ y + 4z = 0 \end{cases}$$

$$\begin{cases} x - 2y + z = 0 \\ -y + 2z = 0 \\ -3y + 2z = 1 \end{cases} \xrightarrow{\text{mult (2, -1)}} \begin{cases} x - 2y + z = 0 \\ y - 2z = 0 \\ -3y + 2z = 1 \end{cases} \xrightarrow{\text{mult (2, 3)}} \begin{cases} x - 2y + z = 0 \\ y - 2z = 0 \\ y - 4z = 1 \end{cases} \xrightarrow{\text{sub (2,1)}} \begin{cases} x - 2y + z = 0 \\ y - 2z = 0 \\ -3z = 1 \end{cases}$$

$$\begin{cases} x - 2y + z = 0 \\ y - 2z = 0 \\ -3z = 1 \end{cases} \xrightarrow{\text{mult (3, -1/3)}} \begin{cases} x - 2y + z = 0 \\ y - 2z = 0 \\ z = 1/4 \end{cases} \xrightarrow{\text{sub (3, 2)}} \begin{cases} x - 3z = 0 \\ y - 2z = 0 \\ z = 1/4 \end{cases} \xrightarrow{\text{sub (3, 1)}} \begin{cases} x = 3/4 \\ y = 1/2 \\ z = 1/4 \end{cases}$$

$$\boxed{x = 3/4 \quad y = 1/2 \quad z = 1/4}$$