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> with(LinearAlgebra):
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> A := <0,1|1,1>;Eigenvectors(A);
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$$A := \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$
$$\begin{bmatrix} \frac{1}{2}\sqrt{5} + \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2}\sqrt{5} \end{bmatrix}, \begin{bmatrix} \frac{1}{2}\sqrt{5} + \frac{1}{2} & \frac{1}{2} - \frac{1}{2}\sqrt{5} \\ 1 & 1 \end{bmatrix} \quad (1)$$

```
> v1:=<1,c>;v2:=<1,d>;D1:=<c,0|0,d>^+;P:=<v1|v2>;
```

$$D1 := \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$$
$$P := \begin{bmatrix} 1 & 1 \\ c & d \end{bmatrix} \quad (2)$$

```
> simplify(subs(c=(1+sqrt(5))/2,d=(1-sqrt(5))/2,A.P-P.D1));
```

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (3)$$

```
> D2:=<c^k,0|0,d^k>^+;
```

$$D2 := \begin{bmatrix} c^k & 0 \\ 0 & d^k \end{bmatrix} \quad (4)$$

```
> Ak:=simplify(P .D2 . (1/P));
```

$$Ak := \begin{bmatrix} -\frac{c^k d - d^k c}{c-d} & \frac{c^k - d^k}{c-d} \\ -\frac{c^{1+k} d - d^{1+k} c}{c-d} & \frac{c^{1+k} - d^{1+k}}{c-d} \end{bmatrix} \quad (5)$$

```
> # Because cd=-1, this reduces to 1/(c-d) times the matrix
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```
> simplify(<-d*c^k+c*d^k,c^k-d^k|c^k-d^k,c*c^k-d*k>^+);
```

$$\begin{bmatrix} -c^k d + d^k c & c^k - d^k \\ c^k - d^k & c^{1+k} - d^{1+k} \end{bmatrix} \quad (6)$$