1. (Chapter 1: 60 points) Consider the system $A\vec{u} = \vec{b}$ with $\vec{u} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ defined by

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 + x_4 &= 2 \\ 4x_1 + 3x_2 + 8x_3 + x_4 &= 4 \\ 6x_1 + 3x_2 + 8x_3 + x_4 &= 2 \end{aligned} \Rightarrow \left[\begin{array}{cccc|c} 2 & 3 & 4 & 1 & 2 \\ 4 & 3 & 8 & 1 & 4 \\ 6 & 3 & 8 & 1 & 2 \end{array}\right]$$

Solve the following parts:

A (a) [10%] Find the reduced row echelon form of the augmented matrix.

A (b) [10%] Identify the **free** variables and the **lead** variables.

A (c) [10%] Display a vector formula for a particular solution $\vec{u}_p$. $\Rightarrow \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

A (d) [10%] Display a vector formula for the homogeneous solution $\vec{u}_h$. $\Rightarrow t_1 \begin{bmatrix} 0 \\ -1/3 \\ 0 \\ 1 \end{bmatrix}$

A (e) [10%] Identify each of **Strang's Special Solutions**. $\Rightarrow \begin{bmatrix} 0 \\ -1/3 \\ 0 \\ 1 \end{bmatrix}$

A (f) [10%] Display the vector general solution $\vec{u}$, using **superposition**. $\Rightarrow \vec{u} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t_1 \begin{bmatrix} 0 \\ -1/3 \\ 0 \\ 1 \end{bmatrix}$

a) $\left[\begin{array}{cccc|c} 2 & 3 & 4 & 1 & 2 \\ 4 & 3 & 8 & 1 & 4 \\ 6 & 3 & 8 & 1 & 2 \end{array}\right] \begin{array}{l} \\ r_2 - 2r_1 \\ r_3 - 3r_1 \end{array} \left[\begin{array}{cccc|c} 2 & 3 & 4 & 1 & 2 \\ 0 & -3 & 0 & -1 & 0 \\ 0 & -6 & -4 & -2 & -4 \end{array}\right] \begin{array}{l} r_1 + r_2 \\ -\frac{1}{3}r_2 \\ r_3 - 2r_2 \end{array} \left[\begin{array}{cccc|c} 2 & 0 & 4 & 0 & 2 \\ 0 & 1 & 0 & 1/3 & 0 \\ 0 & 0 & -4 & 0 & -4 \end{array}\right]$

$\begin{array}{l} (r_1 + r_3)/2 \\ \\ \frac{1}{4}r_3 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right]$

b) lead var. - x_1, x_2, x_3 ; free var. - x_4

c) $x_1 = -1$

$x_2 = -\frac{1}{3}x_4 = -\frac{1}{3}t_1$

$x_3 = 1$

$x_4 = t_1$

$\Rightarrow \vec{x}_h = t_1 \begin{bmatrix} 0 \\ -1/3 \\ 0 \\ 1 \end{bmatrix}_3$, $\vec{x}_p = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \vec{u}_p$

d) $\vec{u}_h = t_1 \begin{bmatrix} 0 \\ -1/3 \\ 0 \end{bmatrix}$