MATH 2270-2 Exam 1 Spring 2017

1. (10 points)
Assume that matrices A and B are $n \times n$, matrix I is the $n \times n$ identity and C^T denotes the transpose of a matrix C.

(a) Give a counter example or explain why it is true. If matrix A is invertible, then $A^T(A^{-1}+B)^T = I + (BA)^T$.

(b) Give a counter example or explain why it is true. If $A^2B^2 = I$, then AB is the inverse of BA.

2. (10 points) Definition: An **elementary matrix** is the answer after applying exactly one combo, swap or multiply to the identity matrix I. An **elimination matrix** is a product of elementary matrices.
Let A be a 3×4 matrix. Find the elimination matrix E which under left multiplication against matrix A performs both (1) and (2) below with one matrix multiply.

(1) Replace Row 3 of A with Row 3 minus Row 1.

(2) Replace Row 3 of A by Row 3 minus 5 times Row 2.

3. (20 points) Let a, b and c denote constants and consider the system of equations

$$\begin{pmatrix} 1 & c & b \\ 2 & b+c & a \\ 1 & b & -a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -a \\ a \\ a \end{pmatrix}$$

Use techniques learned in this course to briefly explain the following facts. Only write what is needed to justify a statement.

(a). The system has a unique solution for $(c-b)(2a-b) \neq 0$.

(b). The system has no solution if $b = 2a$ and $a \neq 0$ (don't explain the other possibilities).

(c). The system has infinitely many solutions if $a = b = c = 0$ (don't explain the other possibilities).

Definition. *Vectors* $\vec{v}_1, \ldots, \vec{v}_k$ *are called* **independent** *provided solving the equation* $c_1\vec{v}_1 + \cdots + c_k\vec{v}_k = \vec{0}$ *for constants* $c_1, \ldots, c_k$ *has the unique solution* $c_1 = \cdots = c_k = 0$. *Otherwise the vectors are called* **dependent**.

4. (20 points) Classify the following set of vectors as Independent or Dependent, using the Pivot Theorem, the Rank Theorem or the definition of independence (above). Details are 75%, answer 25%.

$$\begin{pmatrix}1\\0\\0\\1\\1\end{pmatrix}, \begin{pmatrix}0\\2\\0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\2\\0\\2\\2\end{pmatrix}, \begin{pmatrix}1\\0\\0\\2\\2\end{pmatrix}$$

5. (20 points) Find the vector general solution $\vec{x}$ to the equation $A\vec{x} = \vec{b}$ for

$$A = \begin{pmatrix}1 & 0 & 0 & 4\\4 & 0 & 0 & 1\\6 & 0 & 2 & 0\end{pmatrix}, \quad \vec{b} = \begin{pmatrix}0\\0\\4\end{pmatrix}$$

6. (20 points) Determinant problem, chapter 3.

(a) [30%] Assume given 3×3 matrices A, B. Suppose $E_3E_2E_1A = A^2B^2$ and E_1, E_2, E_3 are elementary matrices representing respectively a combination, a swap and a multiply by 3. Assume $\det(B) = -5$. Let $C = 2A$. Find all possible values of $\det(C)$.

(b) [20%] Determine all values of x for which $(I + C)^{-1}$ exists, where I is the 3×3 identity and $C = \begin{pmatrix}2 & x & -1\\x & 1 & 1\\1 & 0 & -1\end{pmatrix}$.

c) [30%] Let symbols a, b, c denote constants and define

$$A = \begin{pmatrix}1 & 0 & 0 & 0\\1 & -2 & 0 & 0\\a & b & 0 & 1\\1 & c & 1 & 2\end{pmatrix}$$

Apply the adjugate [adjoint] formula for the inverse

$$A^{-1} = \frac{\mathbf{adj}(A)}{|A|}$$

to find the value of the entry in row 3, column 2 of A^{-1}.

End Exam 1.