

## Sample Quiz 7 Solutions

Extra Credit

Problem 3. write the equations as

$$\begin{pmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 45 \\ 40 \\ 55 \\ 50 \end{pmatrix}$$

The vector  $\vec{x}$  is the vector of equilibrium temperatures, located at the grid points of the diagram.

We do swap, combo, mult on the augmented matrix

$$\left( \begin{array}{cccc|c} 4 & -1 & -1 & 0 & 45 \\ -1 & 4 & 0 & -1 & 40 \\ -1 & 0 & 4 & -1 & 55 \\ 0 & -1 & -1 & 4 & 50 \end{array} \right)$$

or alternatively compute the inverse of the coefficient matrix  $A$ :

$$A^{-1} = \begin{pmatrix} 7 & 2 & 2 & 1 \\ 2 & 7 & 1 & 2 \\ 2 & 1 & 7 & 2 \\ 1 & 2 & 2 & 7 \end{pmatrix} \cdot \frac{1}{24}$$

The solution of  $A\vec{x} = \vec{b}$  is always  $\vec{x} = A^{-1}\vec{b}$ , so

$$\vec{x} = \frac{1}{24} \begin{pmatrix} 7 & 2 & 2 & 1 \\ 2 & 7 & 1 & 2 \\ 2 & 1 & 7 & 2 \\ 1 & 2 & 2 & 7 \end{pmatrix} \begin{pmatrix} 45 \\ 40 \\ 55 \\ 50 \end{pmatrix} = \begin{pmatrix} 23.125 \\ 21.875 \\ 25.625 \\ 24.375 \end{pmatrix} = \begin{pmatrix} 185 \\ 175 \\ 205 \\ 195 \end{pmatrix} \cdot \frac{1}{8}$$

Most of this can be done with technology, as the grid size goes up, so does the difficulty of writing  $A, \vec{b}$ .