

Sample Quiz 4

Sample Quiz4 Problem 1. The velocity of a crossbow arrow fired upward from the ground is given at different times in the following table.

| Time t in seconds | Velocity $v(t)$ in ft/sec | Location |
|---------------------|---------------------------|--------------------|
| 0.000 | 50 | Ground |
| 1.413 | 0 | Maximum |
| 2.980 | -45 | Near Ground Impact |



- (a) The velocity can be approximated by a quadratic polynomial

$$v(t) = at^2 + bt + c$$

which reproduces the table data. Find three equations for the coefficients a, b, c . Then solve for them to obtain $a \approx 2.238$, $b \approx -38.55$, $c = 50$.

- (b) Assume a drag model $v' = -32 - \rho v$. Substitute the polynomial answer of (a) into this differential equation, then substitute $t = 0$ and solve for $\rho \approx 0.131$.
- (c) Solve the model $w' = -32 - \rho w$, $w(0) = 50$ with $\rho = 0.131$.
- (d) Compare $w(t)$ and $v(t)$ in a plot. Comment on the plot and what it means.

References. Edwards-Penney sections 2.3, 3.1, 3.2. Course documents on Linear algebraic equations and Newton kinematics.

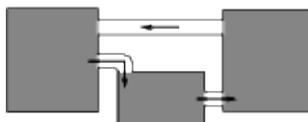
Sample Quiz4 Problem 2. Consider the system of differential equations

$$x_1' = -\frac{1}{6}x_1 + \frac{1}{6}x_3,$$

$$x_2' = \frac{1}{6}x_1 - \frac{1}{3}x_2,$$

$$x_3' = \frac{1}{3}x_2 - \frac{1}{6}x_3,$$

for the amounts x_1, x_2, x_3 of salt in recirculating brine tanks, as in the figure:



Recirculating Brine Tanks A, B, C

The volumes are 60, 30, 60 for A, B, C, respectively.

The steady-state salt amounts in the three tanks are found by formally setting $x_1' = x_2' = x_3' = 0$ and then solving for the symbols x_1, x_2, x_3 . Solve the corresponding linear system of algebraic equations to obtain the answer $x_1 = x_3 = 2c$, $x_2 = c$, which means the total amount of salt is uniformly distributed in the tanks in ratio 2 : 1 : 2.

References. Edwards-Penney sections 3.1, 3.2, 7.3 Figure 5. Course documents on Linear algebraic equations and Systems and Brine Tanks.