Sample Quiz 4

Sample Quiz 4 Problem 1. The velocity of a crossbow arrow fired upward from the ground is given at different times in the following table.

<table>
<thead>
<tr>
<th>Time t in seconds</th>
<th>Velocity v(t) in ft/sec</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>50</td>
<td>Ground</td>
</tr>
<tr>
<td>1.413</td>
<td>0</td>
<td>Maximum</td>
</tr>
<tr>
<td>2.980</td>
<td>-45</td>
<td>Near Ground Impact</td>
</tr>
</tbody>
</table>

(a) The velocity can be approximated by a quadratic polynomial

\[ v(t) = at^2 + bt + c \]

which reproduces the table data. Find three equations for the coefficients \(a, b, c\). Then solve for them to obtain \(a \approx 2.238, b \approx -38.55, c = 50\).

(b) Assume a drag model \(v' = -32 - \rho v\). Substitute the polynomial answer of (a) into this differential equation, then substitute \(t = 0\) and solve for \(\rho \approx 0.131\).

(c) Solve the model \(w' = -32 - \rho w, w(0) = 50\) with \(\rho = 0.131\).

(d) Compare \(w(t)\) and \(v(t)\) in a plot. Comment on the plot and what it means.

References. Edwards-Penney sections 2.3, 3.1, 3.2. Course documents on Linear algebraic equations and Newton kinematics.

Sample Quiz 4 Problem 2. Consider the system of differential equations

\[
\begin{align*}
x_1' &= -\frac{1}{6}x_1 + \frac{1}{6}x_3, \\
x_2' &= \frac{1}{6}x_1 - \frac{1}{3}x_2, \\
x_3' &= \frac{1}{3}x_2 - \frac{1}{6}x_3,
\end{align*}
\]

for the amounts \(x_1, x_2, x_3\) of salt in recirculating brine tanks, as in the figure:

Recirculating Brine Tanks A, B, C

The volumes are 60, 30, 60 for A, B, C, respectively.

The steady-state salt amounts in the three tanks are found by formally setting \(x_1' = x_2' = x_3' = 0\) and then solving for the symbols \(x_1, x_2, x_3\). Solve the corresponding linear system of algebraic equations to obtain the answer \(x_1 = x_3 = 2c, x_2 = c\), which means the total amount of salt is uniformly distributed in the tanks in ratio 2 : 1 : 2.

References. Edwards-Penney sections 3.1, 3.2, 7.3 Figure 5. Course documents on Linear algebraic equations and Systems and Brine Tanks.