Quiz 5

Quiz 5, Problem 1. Harmonic Vibration
A mass of $m = 200$ grams attached to a spring of Hooke’s constant $k$ undergoes free undamped vibration. At equilibrium, the spring is stretched 10 cm by a force of 4 Newtons. At time $t = 0$, the spring is stretched 0.4 m and the mass is set in motion with initial velocity 3 m/s directed away from equilibrium. Find:

(a) The numerical value of Hooke’s constant $k$.

(b) The initial value problem for vibration $x(t)$.

(c) Show details for solving the initial value problem for $x(t)$. The answer is $x(t) = \frac{2}{5} \cos(\sqrt{200}t) + \frac{3}{20} \sqrt{2} \sin(\sqrt{200}t)$, graphed below.
Assume results (a), (b), (c) from Problem 1. In particular, assume

\[ x(t) = \frac{2}{5} \cos(\sqrt{200}t) + \frac{3\sqrt{2}}{20} \sin(\sqrt{200}t). \]

Complete these parts.

(d) Plot the solution \( x(t) \) using technology, approximately matching the graphic below.

(e) Show trig details for conversion of \( x(t) \) to phase-amplitude form

\[ x(t) = \frac{\sqrt{82}}{20} \cos(\sqrt{200}t - \arctan(3\sqrt{2}/8)). \]

(f) Report from the answer in part (e) decimal values for the period, amplitude and phase angle. Two-place decimal accuracy is sufficient.
Quiz 5, Problem 3. Beats

The physical phenomenon of beats refers to the periodic interference of two sound waves of slightly different frequencies. A destructive interference occurs during a very brief interval, so our impression is that the sound periodically stops, only briefly, and then starts again with a beat, a section of sound that is instantaneously loud again. An illustration of the graphical meaning appears in the figure below.

Beats

Shown in red is a periodic oscillation \( x(t) = 2 \sin 4t \sin 40t \) with rapidly–varying factor \( \sin 40t \) and the two slowly–varying envelope curves \( x_1(t) = 2 \sin 4t \) (black), \( x_2(t) = -2 \sin 4t \) (grey).

The undamped, forced spring-mass problem

\( x'' + 1296x = 640 \cos(44t), \; x(0) = x'(0) = 0 \)

has by trig identities the solution

\( x(t) = \cos(36t) - \cos(44t) = 2 \sin 4t \sin 40t. \)

The Problem. Solve the initial value problem

\( x'' + 1444x = 1056 \cos(50t), \; x(0) = x'(0) = 0 \)

by undetermined coefficients and linear algebra, obtaining the solution \( x(t) = \cos(38t) - \cos(50t) \). Then show the trig details for \( x(t) = 2 \sin(6t) \sin(44t) \). Finally, graph \( x(t) \) and its slowly varying envelope curves on \( 0 \leq t \leq \pi \).