Quiz 1. Please prepare your own report on 8x11 paper, handwritten. Work alone or in groups.

Quiz1 Problem 1. An answer check for the differential equation and initial condition

$$\frac{dy}{dx} = k(73 - y(x)), \quad y(0) = 28 \tag{1}$$

requires substitution of the candidate solution $y(x) = 73 - 45\, e^{-kx}$ into the left side (LHS) and right side (RHS), then compare the expressions for equality for all symbols. The process of testing LHS = RHS applies to both the differential equation and the initial condition, making the answer check have **two** presentation panels. Complete the following:

1. Show the two panels in an answer check for initial value problem (1).
2. Relate (1) to a Newton cooling model for warming a 28 F ice cream bar to room temperature 73 F.
3. Let x be the time in minutes. Find the Newton cooling constant k, given the additional information that the ice cream bar reaches 34 F in 5 minutes.

References. Edwards-Penney sections 1.1, 1.4, 1.5. Newton cooling in Serway and Vuille, *College Physics 9/E*, Brooks-Cole (2011), ISBN-10: 0840062060. Newton cooling differential equation $\frac{du}{dt} = -h(u(t) - u_1)$, Math 2280 slide Three Examples. Math 2280 slide on Answer checks.

Quiz1 Problem 2. A 2-ft high conical water urn drains from an orifice 6 inches above the base. The tank drains according to the Torricelli model

$$|y(x)|^2 \frac{dy}{dx} = -0.021\sqrt{|y(x)|}, \quad y(0) = y_0. \tag{2}$$

Symbol $y(x) \ge 0$ is the tank water height in feet above the orifice at time x seconds, while $y_0 \ge 0$ is the water height at time $x = 0$.

Establish these facts about the physical problem.

1. If $y_0 > 0$, then the solution $y(x)$ is uniquely determined and computable by numerical software. Justify using Picard's existence-uniqueness theorem.
2. Solve equation (2) using separation of variables when y_0 is 18 inches, then numerically find the drain time. Check your answer with technology.

References. Edwards-Penney, Picard's theorem 1 section 1.3 and Torricelli's Law section 1.4. Tank draining `Mathematica` demo at Wolfram Research. Carl Schaschke, *Fluid Mechanics: Worked Examples for Engineers*, The Institution of Chemical Engineers (2005), ISBN-10: 0852954980, Chapter 6. Math 2280 slide on Picard and Peano Theorems. Manuscript on applications of first order equations http://www.math.utah.edu/ gustafso/2250SciEngApplications.pdf, Example 35.