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Fractals: Starting From The Base

Table of Contents

Abstract

Fractals are interesting pieces of art made through math. Fractals are theoretically infinite. They are patterns that build on each other. In this case, I will use linear transformations to build fractals. Different transformations will include translation, rotation, reflection, etc. There are many different ways to make fractals, one such example using the Mandelbrot set. The set in particular uses rectangles, and continually becomes smaller and smaller through linear transformations, depending on the number of iterations. Other fractal sets include the Sierpinski carpet, the Mendel sponge, and the Koch curve. Typically, the number of iterations to create fractals is large, potentially going up to the tens or hundreds of thousands.

Fractals are more than just mathematical art. They have their place in science and real world applications. In game design, fractals are used to generate worlds and landscapes. They are also found in biology through nature, with examples like trees and our nervous system. Understanding fractals could help us more in understanding ourselves and the world as an alternative perspective.

Types of Fractals

First, I'll discuss fractals in nature. We can consider many objects in nature as fractals. For example, snowflakes are fractals because of their repeating intricate patterns on the edges. One of the most famous fractal patterns revolves around the snowflake: the Koch curve, which will be discussed later on.

Other examples of real world fractals include trees, ferns lighting, clouds, and many others (Korevaar). An important distinction between real world fractals and mathematical fractals is that real world fractals cannot continue forever, unlike mathematical fractals. Mathematical fractals can be calculated over and over again, into infinity. Let's take a closer look at how fractals are calculated, and 4 famous fractal sets.

How Fractals are Made

Fractals are made through a variety of linear transformations, strung together in two different setups: the similitude and affine transformations.

Similitude

A similitude is an affine transformation with a rotation matrix replacing the general scalar matrix as such:

$$T\left[\begin{pmatrix} x \\ y \end{pmatrix}\right] = s\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

where s, θ, e, and f are adjustable scalars (Klang). This type of transformation would be used where distance ratios need to be preserved ("Similitudes"). That means this maintains the given shape, and what changes is its size.

Affine

An affine transformation is the more general equation above, represented as:

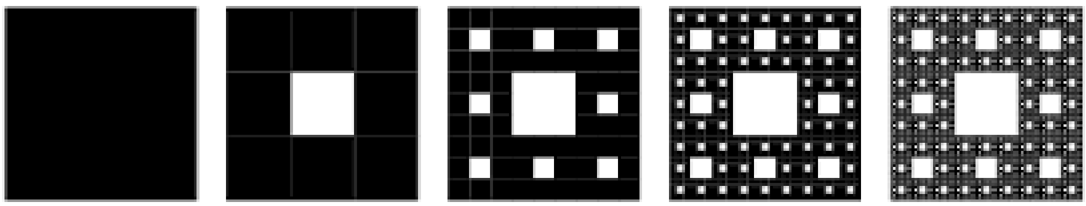
$$T\left[\begin{pmatrix} x \\ y \end{pmatrix}\right] = s\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

where a, b, c, d, and f are scalars (Klang). The 2x2 matrix in the middle can represent a variety of transformation matrices, such as the rotation matrix for the similitude transformation. Instead of rotation, there can be a shear, translation, or scale matrix as a replacement.

Fractal Sets

Sierpinski Carpet

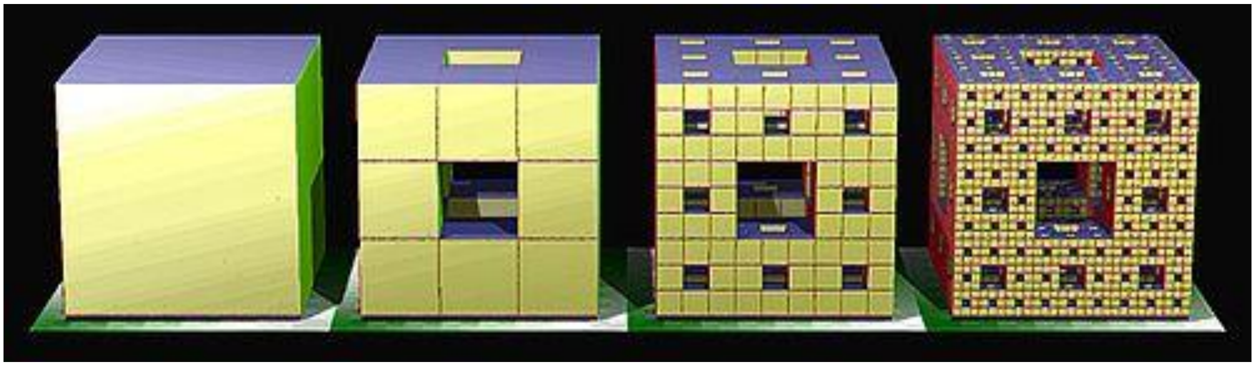
The first fractal set I'll discuss is the Sierpinski carpet, named after Polish mathematician Waclaw Sierpinski (1882-1969). This fractal set transforms a shape 8 times into a square of that shape that outlines the center.



For example, in this representation, the black square transforms 8 times into an outlining square with nothing in the center. The transformation repeats on each of the shapes, repeating to infinity. Another similar set is the Sierpinski Triangle, which uses the same concept as this set, only with 3 transformations in the shape of a triangle instead of 8.

Menger Sponge

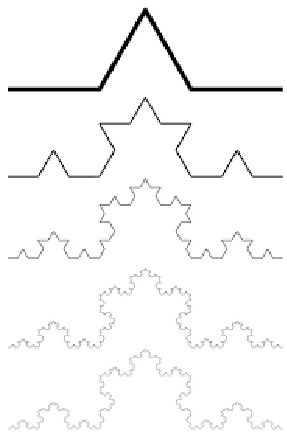
The Menger sponge represents the Sierpinski Carpet in three visible dimensions.



In linear algebra terms, the Sierpinski carpet is represented in R^2, while the Menger sponge is its R^3 representation. The same transformation concepts apply here, except the addition of the third dimension.

Koch Curve

The Koch curve, named after Swedish mathematician Niels van Koch (1870-1924), is a fractal set using translations and rotations to achieve its goal. This set is also known as the Koch snowflake, since after a few iterations, the shape looks like an edge of a snowflake.



This curve is represented by straight lines that divide into 3 partitions. The first and third partitions stay the same. The second partition transforms into two lines of equal length, connecting to each other within the partition. This transformation repeats on each line, up to infinity.

References

Korevaar, Nick. *Fractals*. N.p., n.d. Web. 30 Apr. 2017. <http://www.math.utah.edu/~korevaar/fractals/>.

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