The fifth week finishes the work from chapter 1 and starts chapter 2. Here’s the list of problems, followed by a few answers.

**Section 1.9.** Exercises 15, 25, 31, 39

**Section 2.1.** Exercises 13, 18, 23, 28, 29

**Section 2.2.** Exercises 11, 23, 24, 35

**Section 2.3.** Exercises 7, 13, 14, 17, 19, 20, 21, 33, 35

**Some Answers**

2.1-18: The first two columns of $AB$ are $Ab_1$ and $Ab_2$. They are equal since $b_1$ and $b_2$ are equal.

2.1-28: Since the inner product $u^T v$ is a real number, it equals its transpose. That is, $u^T v = (u^T v)^T = v^T (u^T)^T = v^T u$. Applied here is Theorem 3(d), regarding the transpose of a product of matrices, and Theorem 3(a). The outer product $uv^T$ is an $nn$ matrix. By Theorem 3, $(uv^T)^T = (v^T)^T u^T = vu^T$.

2.2-24: If the equation $Ax = b$ has a solution for each $b$ in $\mathbb{R}^n$, then $A$ has a pivot position in each row, by Theorem 4 in Section 1.4. Since $A$ is square, then the pivots must be on the diagonal of $A$. It follows that $A$ is row equivalent to the identity matrix $I_n$. By Theorem 7, matrix $A$ is invertible.

2.3-14: If $A$ is lower triangular with nonzero entries on the diagonal, then these $n$ diagonal entries can be used as pivots to produce zeros below the diagonal. Thus $A$ has $n$ pivots and so it is invertible, by the Invertible Matrix Theorem. If one of the diagonal entries in $A$ is zero, then $A$ will have fewer than $n$ pivots and hence $A$ will be singular.

2.3-20: By the box following the Invertible Matrix Theorem, $E$ and $F$ are invertible and are inverses. So $FE = I = EF$, and therefore matrices $E$ and $F$ commute.