Linear Algebra 2270-2
Due in Week 2

For the second week, the plan is to start chapter 1. Here’s the list of problems, followed by problem notes and a few answers. The Student Study Guide has answers and solutions to the odd problems. The textbook has answers to the odd problems.

**Problem week2-1.** State the *Three Possibilities* for the solution $\vec{x}$ of a matrix system $A\vec{x} = \vec{b}$. Give an example of each possibility in dimension 3, then describe the geometry of each example.

**Problem week2-2.** In section 2.8 the *nullspace* of a matrix $A$ is defined to be the set of all solutions $\vec{x}$ to the equation $A\vec{x} = \vec{0}$. Give a $3 \times 3$ example in both matrix form and equation form. Then define *nullspace* for a system of equations.

**Section 1.1.** Exercises 7, 11, 19, 25, 34.

**Section 1.2.** Exercises 1, 7, 11, 13, 15, 17, 19, 23, 25, 28, 33.

**Section 1.3.** Exercises 11, 13, 15, 21, 25, 34.

**Some Answers**

Exercise 1.1-34:
1. Begin by interchanging R1 and R4, then create zeros in the first column.
2. Scale R1 by 1 and R2 by 1/4, create zeros in the second column, and replace R4 by R4 + R3.
3. Scale R4 by 1/12, use R4 to create zeros in column 4, and then scale R3 by 1/4.
4. The last step is to replace R1 by R1 + (1)R3.

The solution is $(20, 27.5, 30, 22.5)$.

Exercise 1.2-28: Every column in the augmented matrix except the rightmost column is a pivot column, and the rightmost column is not a pivot column.

Exercise 1.3-34: a. For $j = 1, \ldots, n$, $u_j + (-1)u_j = (-1)u_j + u_j = 0$, by properties of $\mathcal{R}$. By vector equality, $\vec{u} + (-1)\vec{u} = (-1)\vec{u} + \vec{u} = \vec{0}$.

b. For scalars $c$ and $d$, the $j$th entries of $c(d\vec{u})$ and $(cd)\vec{u}$ are $c(du_j)$ and $(cd)u_j$, respectively. These entries in $\mathcal{R}$ are equal, so the vectors $c(d\vec{u})$ and $(cd)\vec{u}$ are equal.