MATH 2270-2 **Sample Exam 1** Spring 2017

1. (10 points)
(a) Give a counter example or explain why it is true. If A and B are $n \times n$ invertible, and C^T denotes the transpose of a matrix C, then $(AB^{-1})^T = (B^T)^{-1}A^T$.
(b) Give a counter example or explain why it is true. If square matrices A and B satisfy $AB = I$, then $BA = I$ and $A^T B^T = I$.

2. (10 points) Let A be a 3×4 matrix. Find the elimination matrix E which under left multiplication against A performs both (1) and (2) with one matrix multiply.

(1) Replace Row 2 of A with Row 2 minus Row 3.

(2) Replace Row 3 of A by Row 3 minus 4 times Row 1.

3. (30 points) Let a, b and c denote constants and consider the system of equations

$$\begin{pmatrix} 1 & b & c \\ 1 & c & -a \\ 2 & b+c & a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -a \\ a \\ a \end{pmatrix}$$

Use techniques learned in this course to briefly explain the following facts. Only write what is needed to justify a statement.

(a). The system has a unique solution for $(c-b)(2a-c) \neq 0$.

(b). The system has no solution if $c = 2a$ and $a \neq 0$ (don't explain the other possibilities).

(c). The system has infinitely many solutions if $a = b = c = 0$ (don't explain the other possibilities).

Definition. *Vectors* $\vec{v}_1, \ldots, \vec{v}_k$ *are called* **independent** *provided solving the equation* $c_1\vec{v}_1 + \cdots + c_k\vec{v}_k = \vec{0}$ *for constants* $c_1, \ldots, c_k$ *has the unique solution* $c_1 = \cdots = c_k = 0$. *Otherwise the vectors are called* **dependent**.

4. (20 points) Classify the following sets of vectors as Independent or Dependent, using the Pivot Theorem or the definition of independence (above).

$$\text{Set 1:} \quad \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$\text{Set 2:} \quad \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\end{pmatrix}, \begin{pmatrix}1\\2\\0\end{pmatrix}$$

5. (20 points) Find the vector general solution $\vec{x}$ to the equation $A\vec{x} = \vec{b}$ for

$$A = \begin{pmatrix}1 & 0 & 0 & 4\\3 & 0 & 1 & 0\\4 & 0 & 0 & 1\end{pmatrix}, \quad \vec{b} = \begin{pmatrix}0\\4\\0\end{pmatrix}$$

6. (20 points) Determinant problem, chapter 3. Parts reduced on Exam 1.
(a) [10%] True or False? The value of a determinant is the product of the diagonal elements.
(b) [10%] True or False? The determinant of the negative of the $n \times n$ identity matrix is -1.
(c) [30%] Assume given 3×3 matrices A, B. Suppose $E_2E_1A^2 = AB$ and E_1, E_2 are elementary matrices representing respectively a combination and a multiply by 3. Assume $\det(B) = 27$. Let $C = -A$. Find all possible values of $\det(C)$.
(d) [20%] Determine all values of x for which $(2I + C)^{-1}$ fails to exist, where I is the 3×3 identity and $C = \begin{pmatrix}2 & x & -1\\3x & 0 & 1\\1 & 0 & -1\end{pmatrix}$.
(e) [30%] Let symbols a, b, c denote constants and define

$$A = \begin{pmatrix}1 & -1 & 0 & 0\\1 & 0 & 0 & 0\\a & b & 0 & 1\\1 & c & 1 & \frac{1}{2}\end{pmatrix}$$

Apply the adjugate [adjoint] formula for the inverse

$$A^{-1} = \frac{\mathbf{adj}(A)}{|A|}$$

to find the value of the entry in row 4, column 2 of A^{-1}.

End Exam 1.