

The No Solution Case

- **No Solution Case**
- **An Example**
- **An Illustration of the No Solution Case**
- **Perplexing Toolkit Sequences**

No Solution Case

A **signal equation** is a nonzero equation having no variables. It is typically encountered in toolkit sequences as the equation $\mathbf{0} = \mathbf{1}$.

When a signal equation occurs in a toolkit sequence, then we report **no solution**, because a signal equation is a false equation, implying that the system of equations cannot have a solution.

An Example

$$\begin{array}{rcl} x + 2y + 3z & = & 4, \\ -y - 3z & = & -1, \\ 0 & = & 1. \end{array}$$

Signal Equation $0 = 1$.

An Illustration of the No Solution Case

$$\begin{array}{rcl} & y + 3z & = 2, \\ x + & y & = 3, \\ x + 2y + 3z & = & 4. \end{array}$$

Frame 1. Original system.

$$\begin{array}{rcl} x + 2y + 3z & = & 4, \\ x + & y & = 3, \\ & y + 3z & = 2. \end{array}$$

Frame 2.

swap(1, 3)

$$\begin{array}{rcl} x + 2y + 3z & = & 4, \\ - & y - 3z & = -1, \\ & y + 3z & = 2. \end{array}$$

Frame 3.

combo(1, 2, -1)

$$\begin{array}{rcl} x + 2y + 3z & = & 4, \\ - & y - 3z & = -1, \\ & 0 & = 1. \end{array}$$

Frame 4.

Signal Equation $0 = 1$.

combo(2, 3, 1)

The signal equation $0 = 1$ is a false equation, therefore the last frame has no solution. Because the toolkit neither creates nor destroys solutions, then the first frame, which is the original system, has **no solution**.

Perplexing Toolkit Sequences

Values cannot be assigned to any variables in the case of no solution. This can be perplexing, especially in a final toolkit step like

$$\begin{array}{l} x = 4, \\ z = -1, \\ 0 = 1. \end{array}$$

While it is true that x and z were assigned values, the final signal equation $0 = 1$ is false, meaning any answer is impossible.

There is no possibility to write equations for all variables. There is **no solution**. It is a **tragic error** to claim $x = 4, z = -1$ is a solution.