Name ____________________________ Class Time ______

Math 2270 Extra Credit Problems
Chapter 5
December 2011

These problems were created for Bretscher's textbook, but apply for Strang's book, except for the division by chapter. To find the background for a problem, consult Bretscher's textbook, which can be checked out from the math library or the LCB Math Center.

Due date: See the internet due dates. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

Problem Xc5.1-10. (Angle)

For which values of k are the vectors $\mathbf{u} = \begin{pmatrix} 2k \\ 1 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix}$ perpendicular?

Problem Xc5.1-26. (Orthogonal Projection)

Find the orthogonal projection of $\mathbf{w}$ onto the subspace V, given

$$\mathbf{w} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad V = \mathbf{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \right\}.$$

Problem Xc5.1-34. (Minimization)

Among all the vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ in $\mathcal{R}^3$, find the one with unit length that minimizes the sum $x + 2y + 3z$.

Problem Xc5.2-14. (Gram-Schmidt Basis)

Given the basis below, labeled $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, find the Gram-Schmidt basis $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$.

$$\begin{pmatrix} 1 \\ 7 \\ 1 \\ 7 \end{pmatrix}, \begin{pmatrix} 0 \\ 7 \\ 2 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 15 \\ 2 \\ 13 \end{pmatrix}.$$

Problem Xc5.2-20. (QR-Factorization)

Find the factorization $M = QR$, given

$$M = \begin{pmatrix} 4 & 25 & 0 \\ 0 & 0 & -2 \\ 3 & -25 & 0 \end{pmatrix}.$$

Problem Xc5.2-34. (Kernel)

Find an orthonormal basis for the kernel of the matrix $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{pmatrix}$.

Problem Xc5.2-38. (QR-Factorization)

Find the factorization $M = QR$, given $M = \begin{pmatrix} 0 & -3 & 0 \\ 0 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

Problem Xc5.3-11. (Orthogonal Matrices)
Given A and B are orthogonal, then which of the following must be orthogonal?
(a) $2A$, (b) ABA, (c) $A^{-1}B^T$, (d) $A - AB$, (e) $AB + BA$, (f) $-BA$

Problem Xc5.3-20. (Symmetric Matrices)
Given A and B are symmetric matrices and A is invertible, then which of the following must also be symmetric?
(a) A^TA, (b) ABA, (c) $A^{-1}B$, (d) $A - B$, (e) $A - BA$, (f) $A - A^T$, (g) A^TB^TBA, (h) $B(A + A^T)B^T$

Problem Xc5.3-26. (Dot Product)
Let T be an orthogonal transformation from $\mathcal{R}^n$ to $\mathcal{R}^n$. Prove that $\mathbf{u} \cdot \mathbf{v} = T(\mathbf{u}) \cdot T(\mathbf{v})$ for all vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathcal{R}^n$.

Problem Xc5.3-32a. (Orthogonal Matrices)
Assume A is $n \times m$ and $A^TA = I$. Is AA^T the identity matrix? Explain.

Problem Xc5.3-44. (Orthogonal Matrices)
Consider an $n \times m$ matrix A. Find in terms of n and m the value of the sum $\mathbf{rank}(\mathbf{dim}(A)) + \mathbf{rank}(\mathbf{ker}(A^T))$.

Problem Xc5.3-50. (QR-Factorization)

(a) Find all square matrices A that are both orthogonal and upper triangular with positive diagonal entries.
(b) Show that the QR-factorization is unique for an invertible square matrix A. Hint: see Exercise 50b in **Bretscher 3E**, section 5.3.

Problem Xc5.4-5. (Basis of $V^\perp$)
Find a basis for $V^\perp$, where $V = \mathbf{ker}(A)$ and

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Problem Xc5.4-16. (Rank)
Prove or disprove: The equation $\mathbf{rank}(A) = \mathbf{rank}(A^TA)$ hold for all square matrices A.

Problem Xc5.4-22. (Least Squares)
Find the least squares solution $\mathbf{x}^*$ of the system $A\mathbf{x} = \mathbf{b}$, given

$$A = \begin{pmatrix} 3 & 2 \\ 5 & 3 \\ 4 & 5 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 10 \\ 18 \\ 4 \end{pmatrix}.$$

Problem Xc5.4-26. (Least Squares)
Find the least squares solution $\mathbf{x}^*$ of the system $A\mathbf{x} = \mathbf{b}$, given

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}.$$

Problem Xc5.5-10. (Orthonormal Basis)
Find an orthonormal basis for $V^\perp$, where $V = \mathbf{span}\{1 + t^2\}$, in the space W of all polynomials $a_0 + a_1t + a_2t^2$ with inner product $< f, g >= \frac{1}{2}\int_{-1}^{1} f(t)g(t)dt$.

Problem Xc5.5-24. (Orthonormal Basis)

Consider the linear space P of all polynomials with inner product $< f, g >= \int_0^1 f(t)g(t)dt$. Let f, g, h be three polynomials satisfying the relations

$$\begin{array}{lll} < f, f >= 4 & < f, g >= 0 & < f, h >= 8 \\ < g, f >= 0 & < g, g >= 2 & < g, h >= 4 \\ < h, f >= 8 & < h, g >= 4 & < h, h >= 10 \end{array}$$

(a) Find $< f, g + 2h >$.

(b) Find $\|g + h\|$.

(c) Find c_1, c_2 satisfying $\mathbf{proj}_{\mathbf{span}\{f,g\}}(h) = c_1 f + c_2 g$.

(d) Find an orthonormal basis for the span of f, g, h expressed as linear combinations of f, g and h.

End of extra credit problems chapter 5.