

.1.12

Statement:

Verify Sub. that each given  $f^u$  is a sol. of the given diff. eq.  
Primes denote derivatives w/ resp. to  $x$

$$x^2 y'' - x y' + 2y = 0$$

$$\begin{aligned} & x^2 y'' \\ & - x y' \\ & + 2y = 0 \dots (1) \end{aligned}$$

Plug (2), (3), (4) into (1):

$$\begin{aligned} & x^2 \left(-\frac{1}{x}\right) \{ \sin(\ln x) + \cos(\ln x) \} \\ & - x \{ \cos(\ln x) - \sin(\ln x) \} \\ & + 2x \cos(\ln x) \\ & = -x \sin(\ln x) - x \cos(\ln x) \\ & \quad - x \cos(\ln x) + x \sin(\ln x) \\ & \quad + 2x \cos(\ln x) \\ & = -x \sin(\ln x) - x \cos(\ln x) \\ & \quad + x \sin(\ln x) - x \cos(\ln x) \\ & \quad \quad + 2x \cos(\ln x) \\ & = 0 \end{aligned}$$

Plug (7), (8), (9) into (1)

$$\begin{aligned} & x^2 \left(\frac{1}{x}\right) \{ \cos(\ln x) - \sin(\ln x) \} \\ & - x \{ \sin(\ln x) + \cos(\ln x) \} \\ & + 2x \sin(\ln x) \\ & = x \cos(\ln x) - x \sin(\ln x) \\ & \quad - x \cos(\ln x) - x \sin(\ln x) \\ & \quad \quad + 2x \sin(\ln x) \\ & = 0 \end{aligned}$$

$\therefore y_1$  &  $y_2$  are sol. of the diff. eq.

$$y_1 = x \cos(\ln x) \dots (2)$$

$$\begin{aligned} y_1' &= \cos(\ln x) + x \{ -\sin(\ln x) \} \frac{1}{x} \\ &= \cos(\ln x) - \sin(\ln x) \dots (3) \end{aligned}$$

$$\begin{aligned} y_1'' &= -\sin(\ln x) \left(\frac{1}{x}\right) - \cos(\ln x) \left(\frac{1}{x}\right) \\ &= -\frac{1}{x} \{ \sin(\ln x) + \cos(\ln x) \} \dots (4) \end{aligned}$$

$$y_2 = x \sin(\ln x) \dots (5)$$

$$\begin{aligned} y_2' &= \sin(\ln x) + x \cos(\ln x) \frac{1}{x} \\ &= \sin(\ln x) + \cos(\ln x) \dots (6) \end{aligned}$$

$$\begin{aligned} y_2'' &= \frac{1}{x} \cos(\ln x) - \frac{1}{x} \sin(\ln x) \\ &= \frac{1}{x} \{ \cos(\ln x) - \sin(\ln x) \} \dots (7) \end{aligned}$$