MATH 2280-001, REVIEW PROBLEMS FOR EXAM 1

Note. Exam 1 is on Wednesday, September 30. It will cover the course material up through homework 3. The corresponding sections of the text are 1.1 thru 15.4, 2.1 thru 2.5 and 3.1 thru 3.3, except multiple roots of the characteristic polynomial. There will be a review session on Tuesday, 8:35-9:25 in LCB 225. Calculators and electronic devices will not be allowed on the exam. You will be allowed one side of an 8.5 by 11 inch sheet of notes. This should not be considered a "practice exam", but the problems here ones that make for a good review, and you should be able to do them. The solutions will be posted by sometime Monday afternoon.

1. Consider the differential equation

$$x' = x^2 - x^4$$

Find the equilibrium solutions. For each one determine whether it is stable, asymtotically stable, or unstable. Sketch the solution curves,

2. A tank contains 60 gallons of pure water. Brine containing 1 lb. of salt per gallon enters the tank at 2 gallons a minute, and the solution (perfectly mixed) leaves the tank at 3 gallons a minute.

a. If $x(t)$ denotes the amount of salt in the tank at time t, derive the equation

$$\frac{dx}{dt} + \left(\frac{3}{60-t}\right) = 2$$

That is, show or explain why x satisfies this differential equation.

b. Find $x(t)$.

3. Find the general solution of the differential equation

$$2y^{(3)} - 2y'' + 13y' = 0$$

4. Consider the differential equation:

$$y'' + 5y' - 6y = f(x)$$

a. Find the general solution of the associated homogeneous equation (when $f = 0$).

b. Suppose that $f(x) = 10e^{-x}$. Show that $y_p(x) = -e^{-x}$ is a particular solution. What is the general solution?

c. As in part b, assume that $f(x) = 10e^{-x}$. Find the solution satisfying the initial conditions $y(0) = 2$ and $y'(0) = 2$.

5. Consider the differntial equation

$$\frac{dy}{dx} = 3x^2(y^2 + 1)$$

a. If we impose the initial condition $y(0) = 0$, is there a unique solution?

b. . Find the solution when $y(0) = 1$.