Math 2250-1 Workout Wednesday Week 3

Name and Unid: _____

1. (10 points) Solve the DE:

$$\frac{dx}{dt} = \frac{1}{t^2}$$

With $x(1) = 1$. If $x(t)$ is a distance, how far does x travel as $t \to \infty$

Solution: This is solved by direct integration

$$\int dx = \int \frac{1}{t^2} dt + C \tag{1}$$

$$x(t) = -\frac{1}{t} + C \tag{2}$$

$$1 = -1 + C \implies C = 2 \tag{3}$$

2. (10 points) Solve the DE:

$$\frac{dx}{dt} = \frac{x}{t} + te^{-t}$$

With $x(1) = 3$.

Solution: This is linear first order:

$$\rho = e^{-\int \frac{1}{t} dt} \tag{4}$$

$$\rho = e^{-\ln(t)} = \frac{1}{t} \tag{5}$$

$$\rho Q = e^{-t} \tag{6}$$

$$\frac{d}{dt}[\rho x] = e^{-t} \tag{7}$$

$$\frac{x}{t} = -e^{-t} + C \tag{8}$$

$$x(t) = Ct - te^{-t} \tag{9}$$

$$C = 4 \tag{10}$$

3. (10 points) Identify the equilibrium points x^* and stability of the following DE. Then solve the DE with $x(0) = 0$:

$$\frac{dx}{dt} = x^2 - 4x + 3$$