Name and Unid: SOLUTION _____

Write your answer in the space provided. Show work for full credit.

1. (10 points) Find the row reduced echelon form of the matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & -4 \end{bmatrix}.$$

Solution:

$$\begin{bmatrix}
1 & 2 & 1 \\
0 & 1 & -1 \\
-1 & 1 & -4
\end{bmatrix}
\xrightarrow{R_1 + R_3 \to R_3}
\begin{bmatrix}
1 & 2 & 1 \\
0 & 1 & -1 \\
0 & 3 & -3
\end{bmatrix}
\xrightarrow{-3R_2 + R_3 \to R_3}
\begin{bmatrix}
1 & 2 & 1 \\
0 & 1 & -1 \\
0 & 1 & -1 \\
0 & 0 & 0
\end{bmatrix}
\xrightarrow{R_1 - 2R_2 \to R_1}
\begin{bmatrix}
1 & 0 & -1 \\
0 & 1 & -1 \\
0 & 0 & 0
\end{bmatrix}$$

2. (10 points) The augmented matrix of a linear system is given in row reduced form by

$$\begin{bmatrix} 1 & 0 & 0 & -2 & 5 \\ 0 & 1 & -2 & -3 & 2 \end{bmatrix}.$$

Find the solution of the linear system.

Solution: The system has infinitely many solutions. The free variables are x_3, x_4 , the leading variables are x_1, x_2 . The solution is

$$x_1 = 5 + 2t$$
 $x_2 = 2 + 2s + 3t$ $x_3 = s$ $x_4 = t$.