Math 2250 Week 3 Quiz

Name and Unid: SOLUTION ________________________________

Write your answer in the space provided. Show work for full credit.

1. (10 points) Solve the first order linear IVP

$$xy'(x) + 2y(x) = 4x^2 \quad y(1) = 2.$$

HINT: $b \ln a = \ln a^b$.

Solution:

1. Rewrite in the form $y'(x) + P(x)y(x) = Q(x)$:

$$y'(x) + \frac{2}{x}y(x) = 4x.$$

So we have $P(x) = 2/x$ and $Q(x) = 4x$.

2. Find the integrating factor $\rho(x)$:

$$\rho(x) = e^{\int P(x)dx} = e^{\int \frac{2}{x}dx} = e^{2\ln|x|} = e^{\ln|x|^2} = e^{\ln(x^2)} = x^2.$$

3. Multiply both sides by $\rho(x)$:

$$x^2y'(x) + 2xy(x) = 4x^3.$$

4. Recognize that the left-hand side is the derivative of a product:

$$\frac{d}{dx}(\rho(x)y(x)) = \frac{d}{dx}(x^2y(x)) = x^2y' + 2xy.$$

5. Integrate both sides:

$$x^2y(x) = \int 4x^3dx = x^4 + C.$$

Solve for $y(x)$:

$$y(x) = x^2 + \frac{C}{x^2}.$$

6. Solve the IVP with $y(1) = 2$:

$$2 = 1 + C \Rightarrow C = 1.$$

The solution is

$$y(x) = x^2 + \frac{1}{x^2}.$$

2. (10 points) Find the partial fraction decomposition of the fraction

$$\frac{3}{(1-x)(x+2)}.$$

Solution: We want

$$\frac{3}{(1-x)(x+2)} = \frac{A}{1-x} + \frac{B}{x+2}.$$

Factoring the right-hand side we have

$$\frac{3}{(1-x)(x+2)} = \frac{A}{1-x} + \frac{B}{x+2} = \frac{A(x+2)+B(1-x)}{(1-x)(x+2)}.$$

The numerators of both fractions have to be the same, thus

$$3 = A(x+2) + B(1-x) = (A-B)x + 2A + B.$$

Since the previous equation has to be true for all x, we find

$$\begin{cases} A - B = 0 \\ 2A + B = 3. \end{cases}$$

The solution to the previous system is $A = B$ and $3B = 3$ that is $A = B = 1$. The partial fraction decomposition is

$$\frac{3}{(1-x)(x+2)} = \frac{1}{1-x} + \frac{1}{x+2}.$$

Check your answer by combining the fraction again.

Alternatively we can use the formula from class

$$\frac{1}{(x+a)(x+b)} = \frac{1}{b-a}\left(\frac{1}{x+a} - \frac{1}{x-b}\right).$$

Here we have

$$\frac{3}{(1-x)(x+2)} = -3\frac{1}{(x-1)(x+2)}$$

that is $a = -1$ and $b = 2$. Plugging into the formula, we find

$$\frac{3}{(1-x)(x+2)} = -3\frac{1}{3}\left(\frac{1}{x-1} - \frac{1}{x+2}\right) = \frac{1}{1-x} + \frac{1}{x+2}.$$