Math 2250   Week 2 Quiz

Name and Unid: SOLUTION _____

Write your answer in the space provided. Show work for full credit.

1. (10 points) Solve the separable initial value problem

$$y' = \frac{2x}{1 + 2y} \quad y(2) = 0.$$

HINT: The solution will be implicit.

Solution:

1. Separate x and y:
$$(1 + 2y)dy = 2x.$$

2. Integrate both sides:

$$\int (1 + 2y)dy = \int 2x\,dx$$
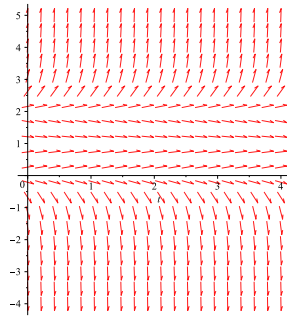
$$\Rightarrow y + \frac{y^2}{2} = x^2 + C.$$

3. Solve the IVP. Plug in $y = 0$ and $x = 2$:

$$0 = 4 + C \Rightarrow C = -4.$$

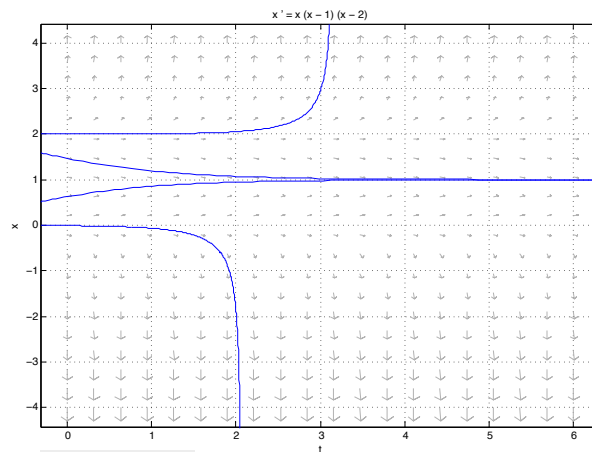The solution to the IVP is

$$2y(x) + y(x)^2 = 2x^2 - 8.$$

2. (10 points) Consider the differential equation $y' = y(y-1)(y-2)$ with the associated slope field given below. This is the logistic equation.

1. On the slope field, sketch the solutions of the IVPs:

$$y(0) = 0.5 \quad \text{and} \quad y(2) = -2 \quad \text{and} \quad y(3) = 3.$$

Solution:

2. From the slope field, what can you say about the behavior as $x \to \infty$ of the solution with $y(0) = 1.5$? In other words, find

$$\lim_{x \to \infty} y(x).$$

Solution:
$$\lim_{y \to \infty} y(x) = 1.$$