Quiz 1. Please prepare your own report on 8x11 paper, handwritten. Work alone or in groups.

Quiz1 Problem 1. An answer check for the differential equation and initial condition

\[
\frac{dy}{dx} = k(73 - y(x)), \quad y(0) = 28
\]  

requires substitution of the candidate solution \( y(x) = 73 - 45e^{-kx} \) into the left side (LHS) and right side (RHS), then compare the expressions for equality for all symbols. The process of testing LHS = RHS applies to both the differential equation and the initial condition, making the answer check have two presentation panels. Complete the following:

1. Show the two panels in an answer check for initial value problem (1).
2. Relate (1) to a Newton cooling model for warming a 28 F ice cream bar to room temperature 73 F.
3. Let \( x \) be the time in minutes. Find the Newton cooling constant \( k \), given the additional information that the ice cream bar reaches 34 F in 5 minutes.


Quiz1 Problem 2. A 2-ft high conical water urn drains from an orifice 6 inches above the base. The tank drains according to the Torricelli model

\[
|y(x)|^2 \frac{dy}{dx} = -0.021 \sqrt{|y(x)|}, \quad y(0) = y_0.
\]  

Symbol \( y(x) \geq 0 \) is the tank water height in feet above the orifice at time \( x \) seconds, while \( y_0 \geq 0 \) is the water height at time \( x = 0 \).

Establish these facts about the physical problem.

1. If \( y_0 > 0 \), then the solution \( y(x) \) is uniquely determined and computable by numerical software. Justify using Picard’s existence-uniqueness theorem.
2. Solve equation (2) using separation of variables when \( y_0 \) is 18 inches, then numerically find the drain time. Check your answer with technology.