

```
libname := "/u/ma/gustafson/bin/laylinalg.mla",
"/usr/local/sys/maple/maple2016/lib" (1)
```

```
> # Solve for the exact answers
# System x'=3x-2y, y'=5x-4y, x(0)=3, y(0)=6
de1:=diff(x(t),t)=3*x(t)-2*y(t);de2:=diff(y(t),t)=5*x(t)-4*y(t);
ic:=x(0)=3,y(0)=6;
dsolve([de1,de2,ic],[x(t),y(t)]);
```

$$de1 := \frac{d}{dt} x(t) = 3 x(t) - 2 y(t)$$

$$de2 := \frac{d}{dt} y(t) = 5 x(t) - 4 y(t)$$

$$ic := x(0) = 3, y(0) = 6$$

$$\{x(t) = 2 e^{-2t} + e^t, y(t) = 5 e^{-2t} + e^t\} \quad (2)$$

```
> Ans:=t->evalf(<2*exp(-2*t)+exp(t), 5*exp(-2*t)+exp(t)>); # Exact
answers
```

$$Ans := t \rightarrow evalf(\langle 2 e^{-2t} + e^t, 5 e^{-2t} + e^t \rangle) \quad (3)$$

```
> # Approximate methods are Euler, Heun, RK4.
# Notation for t-values: t0=initial point, t1=t0+h, t2=t1+h, ...
# Notation for u-values: 1u0=u(t0)=initial values, u1 approx u
(t1), u2 approx u(t2), ....
```

```
> # Solve for approximate answers using Euler's Method
F:=(t,u)-><3*u[1]-2*u[2],5*u[1]-4*u[2]>;
```

$$F := (t, u) \rightarrow \langle 3 u_1 - 2 u_2, 5 u_1 - 4 u_2 \rangle \quad (4)$$

```
> h:=0.1; t0:=0; u0:=<3,6>;
```

$$h := 0.1$$

$$t0 := 0$$

$$u0 := \begin{bmatrix} 3 \\ 6 \end{bmatrix} \quad (5)$$

```
> u1:=u0+h*F(t0,u0);t1:=t0+h;Ans(t1);
```

$$u1 := \begin{bmatrix} 2.700000000000000 \\ 5.100000000000000 \end{bmatrix}$$

$$t1 := 0.1$$

$$\begin{bmatrix} 2.742632424 \\ 5.198824684 \end{bmatrix} \quad (6)$$

```
> u2:=u1+h*F(t1,u1);t2:=t1+h;Ans(t2);
```

$$u2 := \begin{bmatrix} 2.490000000000000 \\ 4.410000000000000 \end{bmatrix}$$

$$t2 := 0.2$$

$$\begin{bmatrix} 2.562042850 \\ 4.573002988 \end{bmatrix} \quad (7)$$

> # Solve for approximate answers using Heun's Method

h:=0.1; t0:=0; u0:=<3,6>;

$$h := 0.1$$

$$t0 := 0$$

$$u0 := \begin{bmatrix} 3 \\ 6 \end{bmatrix} \quad (8)$$

> w:=u0+h*F(t0,u0);t1:=t0+h;u1:=u0+h*(F(t0,u0)+F(t1,w))/2;Ans(t1);

$$w := \begin{bmatrix} 2.700000000000000 \\ 5.100000000000000 \end{bmatrix}$$

$$t1 := 0.1$$

$$u1 := \begin{bmatrix} 2.745000000000000 \\ 5.205000000000000 \end{bmatrix}$$

$$\begin{bmatrix} 2.742632424 \\ 5.198824684 \end{bmatrix} \quad (9)$$

> w:=u1+h*F(t1,u1);t2:=t1+h;u2:=u1+h*(F(t1,u1)+F(t2,w))/2;Ans(t2);

$$w := \begin{bmatrix} 2.527500000000000 \\ 4.495500000000000 \end{bmatrix}$$

$$t2 := 0.2$$

$$u2 := \begin{bmatrix} 2.565825000000000 \\ 4.583025000000000 \end{bmatrix}$$

$$\begin{bmatrix} 2.562042850 \\ 4.573002988 \end{bmatrix} \quad (10)$$

> # Solve for approximate answers using the RK4 Method

h:=0.1; t0:=0; u0:=<3,6>;

$$h := 0.1$$

$$t0 := 0$$

$$u0 := \begin{bmatrix} 3 \\ 6 \end{bmatrix} \quad (11)$$

> k1:=h*F(t0,u0);t1:=t0+h;
k2:=h*F(t0+h/2,u0+k1/2);

```

k3:=h*F(t0+h/2,u0+k2/2);
k4:=h*F(t1,u0+k3);
u1:=u0+(k1+2*k2+2*k3+k4)/6;Ans(t1);

```

$$k1 := \begin{bmatrix} -0.3000000000000000 \\ -0.9000000000000000 \end{bmatrix}$$

$$t1 := 0.1$$

$$k2 := \begin{bmatrix} -0.2550000000000000 \\ -0.7950000000000000 \end{bmatrix}$$

$$k3 := \begin{bmatrix} -0.2587500000000000 \\ -0.8047500000000000 \end{bmatrix}$$

$$k4 := \begin{bmatrix} -0.2166750000000000 \\ -0.7074750000000000 \end{bmatrix}$$

$$u1 := \begin{bmatrix} 2.742637500000000 \\ 5.198837500000000 \end{bmatrix}$$

$$\begin{bmatrix} 2.742632424 \\ 5.198824684 \end{bmatrix}$$

(12)

```

> k1:=h*F(t1,u1);t2:=t1+h;
k2:=h*F(t1+h/2,u1+k1/2);
k3:=h*F(t1+h/2,u1+k2/2);
k4:=h*F(t1,u1+k3);
u2:=u1+(k1+2*k2+2*k3+k4)/6;Ans(t2);

```

$$k1 := \begin{bmatrix} -0.2169762500000000 \\ -0.7082162500000000 \end{bmatrix}$$

$$t2 := 0.2$$

$$k2 := \begin{bmatrix} -0.1787010625000000 \\ -0.6208170625000000 \end{bmatrix}$$

$$k3 := \begin{bmatrix} -0.1816997031250000 \\ -0.6287281031250000 \end{bmatrix}$$

$$k4 := \begin{bmatrix} -0.1457405403125000 \\ -0.5475748603125000 \end{bmatrix}$$

$$u2 := \begin{bmatrix} 2.56205111307292 \\ 4.57302392640625 \end{bmatrix}$$

[

$$\begin{bmatrix} 2.562042850 \\ 4.573002988 \end{bmatrix}$$

(13)