

Name _____ Class Time _____

Project 7. Solve problems L7-1 to L7-5. The problem headers:

----- PROBLEM L7.1. EARTHQUAKE MODEL FOR A BUILDING.
----- PROBLEM L7.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
----- PROBLEM L7.3. UNDETERMINED COEFFICIENTS STEADY-STATE SOL
----- PROBLEM L7.4. PRACTICAL RESONANCE.
----- PROBLEM L7.5. EARTHQUAKE DAMAGE.

FIVE FLOOR Model.

Refer to the textbook of Edwards-Penney, section 5.4 Application (after the section 5.4 exercises). Consider a building with five floors each weighing 50 tons. Each floor corresponds to a restoring Hooke's force with constant $k=5$ tons/foot. Assume that ground vibrations from the earthquake are modeled by $(1/4)\cos(\omega t)$ with period $T=2\pi/\omega$.

PROBLEM L7-1. BUILDING MODEL FOR AN EARTHQUAKE.

Model the 5-floor problem in Maple.

Define the 5 by 5 mass matrix M and Hooke's matrix K for this system and convert $Mx''=Kx$ into the system $x''=Ax$ where A is defined by textbook equation (1), section 5.4 Application.

Sanity check: Mass $m=3125$, and the 5x5 matrix contains fraction $16/5$.

Then find the eigenvalues of the matrix A to six digits, using the Maple command `"linalg[eigenvals](A)."`

Sanity check: All six eigenvalues should be negative.

```
# Sample Maple code for a model with 4 floors.  
# Use maple help to learn about evalf and eigenvals.  
# A:=matrix([ [-20,10,0,0], [10,-20,10,0],  
[0,10,-20,10],[0,0,10,-10]]);  
# with(linalg): evalf(eigenvals(A));
```

```
# Problem L7.1
```

```
# Define k, m and the 5x5 matrix A.
```

```
# with(linalg): evalf(eigenvals(A));
```

PROBLEM L7-2. TABLE OF NATURAL FREQUENCIES AND PERIODS.

Refer to figure 5.4.17 in Edwards-Penney.

Find the natural angular frequencies $\omega=\sqrt{-\lambda}$ for the five story building and also the corresponding periods $2\pi/\omega$, accurate to six digits. Display the answers in a table. Compare with answers in Figure 5.4.17 (actually a table), for the 7-story case.

```
# Sample code for a 4x3 table, 4-story building.  
# Use maple help to learn about nops and printf.  
# ev:=[-10,-1.206147582,-35.32088886,-23.47296354]: n:=nops(ev):
```

```

# Omega:=lambda -> sqrt(-lambda):
# format:="%10.6f %10.6f %10.6f\n":
# seq(sprintf(format,ev[i],Omega(ev[i]),2*evalf(Pi)/Omega(ev[i])),
i=1..n);

# Problem L7-2
# ev:=[fill this in]: n:=nops(ev):
# Omega:=lambda -> sqrt(-lambda): format:="%10.6f %10.6f %10.6f\n":
# seq(sprintf(format,ev[i],Omega(ev[i]),2*evalf(Pi)/Omega(ev[i])),i=1..n);

```

PROBLEM L7-3. UNDETERMINED COEFFICIENTS
STEADY-STATE PERIODIC SOLUTION.

Consider the forced equation $x'=Ax+\cos(\omega t)b$ where b is a constant vector. The earthquake's ground vibration is accounted for by the extra term $\cos(\omega t)b$, which has period $T=2\pi/\omega$. The solution $x(t)$ is the 5-vector of excursions from equilibrium of the corresponding 5 floors. Sought here is not the general solution, which certainly contains transient terms, but rather the steady-state periodic solution, which is known from the theory to have the form $x(t)=\cos(\omega t)c$ for some vector c that depends only on A and b .

Define $b:=0.25*\omega*\omega*\text{vector}([1,1,1,1,1])$: in Maple and find the vector c in the undetermined coefficients solution $x(t)=\cos(\omega t)c$. Vector c depends on ω . As outlined in the textbook, vector c can be found by solving the linear algebra problem $-\omega^2 c = Ac + b$; see equation (32), section 5.4. Don't print c , as it is too complex; instead, print $c[1]$ as an illustration.

```

# Sample code for defining b and A, then solve for c, 4-floor case.
# See maple help to learn about vector and linsolve.
# w:='w': u:=w*w: b:=0.25*u*vector([1,1,1,1]):
# A:=matrix([ [-20,10,0,0], [10,-20,10,0],
#            [0,10,-20,10],[0,0,10,-10]]);
# Au:=evalm(A+u*diag(1,1,1,1));
# c:=linsolve(Au,-b):
# evalf(c[1],2);

```

```

# PROBLEM L7-3
# Define w, u, b, A, Au, c
# evalf(c[1],2);

```

PROBLEM L7-4. PRACTICAL RESONANCE.

Consider the forced equation $x'=Ax+\cos(\omega t)b$ of L7-3 above with $b:=0.25*\omega*\omega*\text{vector}([1,1,1,1,1])$. Practical resonance can occur if a component of $x(t)$ has large amplitude compared to the vector norm of b . For example, an earthquake might cause a small 3-inch excursion on level ground, but the building's floors might have 50-inch excursions, enough to destroy the building.

Let $\text{Max}(c)$ denote the maximum modulus of the components of vector c . Plot $g(T)=\text{Max}(c(\omega))$ with $\omega=(2\pi)/T$ for periods $T=1$ to $T=5$, ordinates $\text{Max}=0$ to $\text{Max}=10$, the vector $c(\omega)$ being the answer produced in L7.3 above. Compare your figure to the textbook Figure 5.4.18.

```

# Sample maple code to define the function Max(c), 4-floor building.
# Use maple help to learn about norm, vector, subs and linsolve.
# with(linalg):
# w:='w': Max:= c -> norm(c,infinity); u:=w*w:
# b:=0.25*w*w*vector([1,1,1,1]):
# A:=matrix([ [-20,10,0,0], [10,-20,10,0], [0,10,-20,10],
[0,0,10,-10]]);
# Au:=evalm(A+u*diag(1,1,1,1));
# C:=ww -> subs(w=ww,linsolve(Au,-b)):
# plot(Max(C(2*Pi/r)),r=1..5,0..10,numpoints=150);

```

```

# PROBLEM L7.4. WARNING: Save your file often!!!
# w:='w': Max:= c -> norm(c,infinity): u:=w*w:
# Define b
# Define A
# Define Au
# Define C
# plot(Max(C(2*Pi/r)),r=1..5,0..10,numpoints=150);

```

PROBLEM L7.5. EARTHQUAKE DAMAGE.

The maximum amplitude plot of L7-4 can be used to detect the of earthquake damage for a given ground vibration of period T . A ground vibration $(1/4)\cos(\omega t)$, $T=2\pi/\omega$, will be assumed, as in L7-4.

- Replot the amplitudes in L7-4 for periods 1.5 to 5.5 and amplitudes 5 to 10. There will be several spikes.
- Create several zoom-in plots, one for each spike, choosing a T -interval that shows the full spike.
- Determine from the several zoom-in plots approximate intervals for the period T such that some floor in the building will undergo excursions from equilibrium in excess of 5 feet.

```

# Example: Zoom-in on a spike for amplitudes 5 feet to 10 feet,
# periods 1.97 to 2.01. This example for the 4-floor problem.
# with(linalg): w:='w': Max:= c -> norm(c,infinity); u:=w*w:
# Au:=matrix([[-20+u,10,0,0], [10,-20+u,10,0], [0,10,-20+u,10], [0,0,10,-10+u]]);
# b:=0.25*w*w*vector([1,1,1,1]):
# C:=ww -> subs(w=ww,linsolve(Au,-b)):
# plot(Max(C(2*Pi/r)),r=1.97..2,01,5..10,numpoints=150);

```

```

# PROBLEM L7-5. WARNING: Save your file often!!
#(a) Re-plot the five spikes.
# plot(Max(C(2*Pi/r)),r=1.5..5.5,5..10,numpoints=150);
#(b) Plot five zoom-in graphs.
# one:=1.79..1.83:plot(Max(C(2*Pi/r)),r=one,5..10,numpoints=150);
# two:=???:plot(Max(C(2*Pi/r)),r=two,5..10,numpoints=150);
# three:=???:plot(Max(C(2*Pi/r)),r=three,5..10,numpoints=150);
# four:=???:plot(Max(C(2*Pi/r)),r=four,5..10,numpoints=150);
# five:=???:plot(Max(C(2*Pi/r)),r=five,5..10,numpoints=150);
#(c) Print period ranges.
# PeriodRanges:=[one,two,three,four,five];

```