Name ___________________________

Math 2280 Extra Credit Maple Lab 4, Mechanical Oscillations.
Due at the end of the semester, Spring 2016

Due date: See the internet due dates. Maple lab 4 has four problems: L4-1, L4-2, L4-3. Please answer the questions A, B, C , . . . associated with each problem.

Problem scores for the Spring 2016 version of Extra Credit Maple Lab 4, Mechanical Vibrations:

________ L4-1. [100] Under-Damped Free Vibration.
________ L4-2. [100] Undamped Forced Vibration.
________ L4-3. [100] Practical Resonance.

L4-1. Under-Damped Free Vibration

Free Vibrations. Consider the problem of free linear vibrations of a damped spring-mass system

$$mx'' + cx' + kx = 0,$$
$$x(0) = 0, \quad x'(0) = 1,$$
$$m = 4, \quad c = 3.$$

Symbols m, c and k are non-negative constants, representing the mass, viscous damping constant and Hooke's constant, respectively. The under-damped case is studied here, $c^2 < 4km$, as explained in section 5.4 the Edwards-Penney textbook *Differential Equations and Linear Algebra, 3/E*. Inserting $m = 4$, $c = 3$ gives the requirement $9 < 16k$ on Hooke's constant k.

Problem L4-1.

- **A**. Select and display a positive Hooke's constant k so that the solution x(t) is under-damped. Choose the specific value of k so that the graphic in part B below displays well. Check that $x(t) = 0$ for infinitely many t¿0 (the solution oscillates). Display the exact solution $x(t)$ obtained by Maple methods as in the example below.
- **B**. Plot the exact symbolic solution $x(t)$ on a suitable t-interval. Check the graphic against Figure 5.4.9 in Edwards-Penney.
- **C**. Estimate from the graph the decimal value of the pseudo-period. Display the graphical estimate and also the exact pseudo-period $2\pi/w$, where w is the natural frequency of the trigonometric term in the solution $x(t)$ found above in item L4.1-A.

```
# EXAMPLE(Wrong parameters! Change it!)
# Use semicolons to see what you have done.
# Define the differential equation
de:=3*diff(x(t),t,t)+1.5*diff(x(t),t)+4*x(t)=0:
# Solve the characteristic equation.
solve(3*r^2+1.5*r+4=0,r);
# Define the initial conditions
ic:=x(0)=0,D(x)(0)= 1:
# Symbolically solve for x(t)
p:=dsolve({de,ic},x(t),method=laplace):
# Capture the dsolve symbolic solution as a function X(t)
X:=unapply(rhs(p),t):
# Plot the solution
plot(X(t),t=0..5);
```

Maple tip: Click with the mouse on the graphic to print the cursor location (left upper corner of the maple window). The coordinates printed are of the form (x, y). From this coordinate information, a subtraction estimates the period.

L4-2. Undamped Forced Vibration

Forced Linear Vibrations. Consider the undamped ($c = 0$) forced vibration problem for a spring-mass system:

$$mx'' + kx = 5\cos(wt),$$
$$x(0) = 0, \quad x'(0) = 0,$$
$$m = 5, \quad k = 3.5$$

Symbol w is a positive constant, the input natural frequency. Symbols m, k are respectively mass and Hooke's constant.

- **A**. Divide the differential equation by $m = 5$ to obtain $x'' + w_0^2 x = \cos(wt)$ where $w_0^2 = k/m = 3.5/5$ defines the natural angular frequency $w_0 = \sqrt{35/50} = 0.8366600265$. Choose the input natural frequency w to be 3 times larger than the natural angular frequency w_0. Solve for $x(t)$ using Maple's `dsolve()`.

B. Because $w = 3w_0$, then $w \neq w_0$. The solution $x(t)$ is the sum of two functions, one of period $2\pi/w$ and the other of period $2\pi/w_0$ (phenomenon of Beats). Graph the slowly-varying envelope curves and the rapidly-varying solution curve $x(t)$ on a suitable interval. See Figure 3.6.3 in Edwards-Penney.

C. Suggest a value for the forcing frequency w so that the vibration exhibits resonance. Show the resonant behavior in a graphic. Check against Figure 3.6.4 in Edwards-Penney.

L4-3. Practical Resonance

Consider the damped forced vibration problem

$$\begin{aligned}&mx'' + cx' + kx = 5\cos(wt),\\ &x(0) = 0, \quad x'(0) = 0,\\ &m = 4, \quad k = 41.\end{aligned}$$

Symbol w is a positive constant, the input natural frequency. Symbols m, c, k are respectively mass, viscous damping constant and Hooke's constant.

A. Consider the damping constants $c = 2$, $c = 1$ and $c = 1/2$. Compute the amplitude function $C(w)$ [section 5.6] for these three equations, then plot for $w = 0$ to $w = 20$ the three amplitude graphs on a single set of axes. Compare against Figure 3.6.9 in Edwards-Penney (it has one curve, yours has 3 curves).

B. For each case $c = 2$, $c = 1$, $c = 1/2$, print the values w^*, C^* where $C^* = C(w^*) = \max\{C(w) \ : \ 0 \leq w \leq 20\}$. The three data pairs should show that C* becomes larger as c tends to zero.

Maple Hint: Use Maple's mouse interface on the graphic of Part A. Specifically, click on a possible maximum (horizontal tangent) in the graph to display the values w^*, C^* on the screen. Look around the screen to see where maple printed the x, y-coordinates! Copy the values into your maple worksheet report.

```
#EXAMPLE(Beware! Wrong values!)
F:=15: m:=1: k:=25:
c:='c': w:='w':
C:=(w,c)->F/sqrt((k-m*w*w)^2+(c*w)^2):
plot({C(w,4),C(w,3),C(w,2)},w=0..15,color=black);
```