Answer Key:

Name. Dakotah Bean Midterm 2 2280 8:35

5. (ch7)

(a) [25%] Solve $\mathcal{L}(f(t)) = \dfrac{10}{(s^2+8)(s^2+4)}$ for $f(t)$.

(b) [25%] Solve for $f(t)$ in the equation $\mathcal{L}(f(t)) = \dfrac{s+1}{s^2(s+2)}$.

(c) [20%] Solve for $f(t)$ in the equation $\mathcal{L}(f(t)) = \dfrac{s-1}{s^2+2s+5}$.

(d) [10%] Solve for $f(t)$ in the relation

$$\mathcal{L}(f) = \frac{d}{ds}\mathcal{L}(t^2 \sin 3t)$$

(e) [10%] Solve for $f(t)$ in the relation

90%

$$\mathcal{L}(f) = \left(\mathcal{L}\left(t^3 e^{9t} \cos 8t\right)\right)\Big|_{s \to s+3}.$$

1) $\dfrac{10}{(s^2+8)(s^2+4)} = \dfrac{As+B}{s^2+8} + \dfrac{Cs+D}{s^2+4}$

$10 = (As+B)(s^2+4) + (Cs+D)(s^2+8)$

$10 = As^3 + 4As + Bs^2 + 4B + Cs^3 + 8Cs + Ds^2 + 8D$

$$\begin{cases} 0 = As^3 + Cs^3 \\ 0 = Bs^2 + Ds^2 \\ 0 = 4As + 8Cs \\ 10 = 4B + 8D \end{cases} \quad \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{Bmatrix}$$

$$A^{-1}B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 4 & 0 & 8 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix} = \begin{array}{cc} \begin{bmatrix} 0 \\ -5/2 \\ 0 \\ 5/2 \end{bmatrix} & \begin{matrix} A \\ B \\ C \\ D \end{matrix} \end{array}$$

$$f(t) = \mathcal{L}^{-1}\left(\frac{-5/2}{s^2+8} + \frac{5/2}{s^2+4}\right) = \boxed{-\frac{5\sqrt{8}}{16} \sin\sqrt{8}\,t + \frac{5}{4}\sin 2t}$$

(where

$-\dfrac{5}{2} \cdot \dfrac{1}{\sqrt{8}}\left(\dfrac{\sqrt{8}}{s^2+8}\right)$

and $\dfrac{1}{2} \cdot \dfrac{5}{2}\left(\dfrac{2}{s^2+4}\right)$)

Use this page to start your solution. Attach extra pages as needed, then staple.

b)

$$\frac{s+1}{s^2(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2}$$

$$s+1 = As(s+2) + B(s+2) + Cs^2$$

$$s+1 = As^2 + 2As + Bs + 2B + Cs^2$$

$$0 = As^2 + Cs^2$$
$$1 = 2As + Bs$$
$$1 = 2B$$

$$A^{-1}B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{array}{cc} \begin{bmatrix} 1/4 \\ 1/2 \\ -1/4 \end{bmatrix} & \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$$

$$\mathcal{L}^{-1}\left(\frac{1/4}{s} + \frac{1/2}{s^2} - \frac{1/4}{s+2} \right) = 1/4 + 1/2\, t - 1/4\, e^{-2t}$$
$$= \boxed{1/4\left(1 + 2t - e^{-2t}\right)}$$

c)

$$\frac{s-1+1-1}{(s+1)^2+4} = \frac{s+1}{(s+1)^2+4} - \frac{2}{(s+1)^2+4}$$

$$\mathcal{L}^{-1}\left(\frac{s+1}{(s+1)^2+4} - \frac{2}{(s+1)^2+4} \right) = e^{-t}\cos 2t - e^{-t}\sin 2t$$
$$= \boxed{e^{-t}(\cos 2t - \sin 2t)}$$

d) Rule: $\mathcal{L}((-t)f(t)) = \frac{d}{ds}\mathcal{L}(f(t))$

$$\Rightarrow \mathcal{L}^{-1}(\mathcal{L}(f)) = \boxed{f(t) = -t^3 \sin 3t}$$

e) Rule: $\mathcal{L}(f(t))\big|_{s \to s+a} = \mathcal{L}(e^{-at} f(t))$

$$f(t) = e^{-3t}\, t^{?}\, e^{9t} \cos 8t$$

$$\boxed{f(t) = t^3 e^{6t} \cos 8t}$$